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# WAVELET ANALYSIS AS THE METHOD OF TIME SERIES PREPROCESSING FOR SOLVING CLASSIFCATION PROBLEM

**Abstract.** In the article described the difference of Fourier and wavelet transforms. There is the algorithm of continuous wavelet analysis with design formulas. In the article described the calculation of the boundary values for continuous wavelet transform grid.

**Key words:** wavelet analysis, Fourier analysis, continuous wavelet transform, classification problem, signal processing.

## History of wavelet analysis

Wavelet analysis is one of the youngest methodology of analysis of non-stationary signals. Definition of the “Wavelet"' initially was given by A.Grossman and J.Morlet in early 80's of 20's century in the works of analysis seismic signals. Near the 90's this methodology of signal analysis was widely discussed in scientific community and was obtained some theoretical results. The result of theoretical research mostly enclose of modification of wavelet transform for speeding up the calculation of spectrogram. In that period the algorithm of the discrete wavelet transform called Mallat algorithm or fast wavelet transform algorithm was developed.

By the beginning of the 1990s, wavelet analysis was widely used in such areas as signal analysis, image recognition and image synthesis, encryption and decryption of information and others.

## Fourier analysis versus wavelet analysis

The main difference between Fourier transform and wavelet transform is time localization. Lack of time localization made it impossible to analyze non-stationary signals. Mainly for solving this problem the theory of spectral analysis made step from frequency analysis (Fourier transform) to time-frequency analysis (wavelet-transform) [1].

The advantage of this approach is the ability to detect changes in the frequency spectrum that occur over time, which is relevant for non-stationary signals. In the case of a non-stationary signal, the wavelet transform is a more informative than the Fourier transform, and importantly, it allows solving the classification problem with greater accuracy. Such results are achieved due to the presence of an additional degree of freedom in the form of localization in time.

The main difference of the Fourier transform versus wavelet-transform can be demonstrated by the function (1).

$$y(x)=\begin{cases}\cos(10\cdot x), & x\in[0,0.5)\\ \cos(100\cdot x), & x\in(0.5,1]\end{cases} \qquad (1).$$

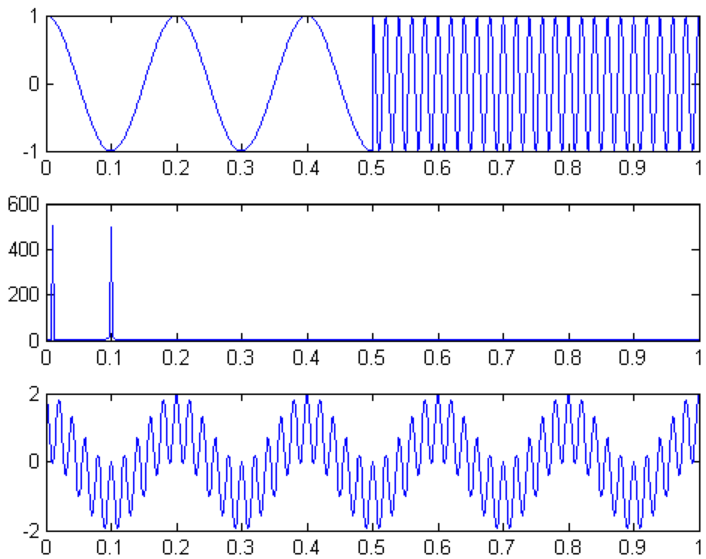


*Fig.1.* Fourier transform of function (1). The first plot is the signal, the second is the Fourier transform of the signal and the third is the inverse Fourier transform.

Source: designed by authors.

The Fourier transform of the function (1) is shown on fig.1 and the wavelet-transform - on the fig. 2.

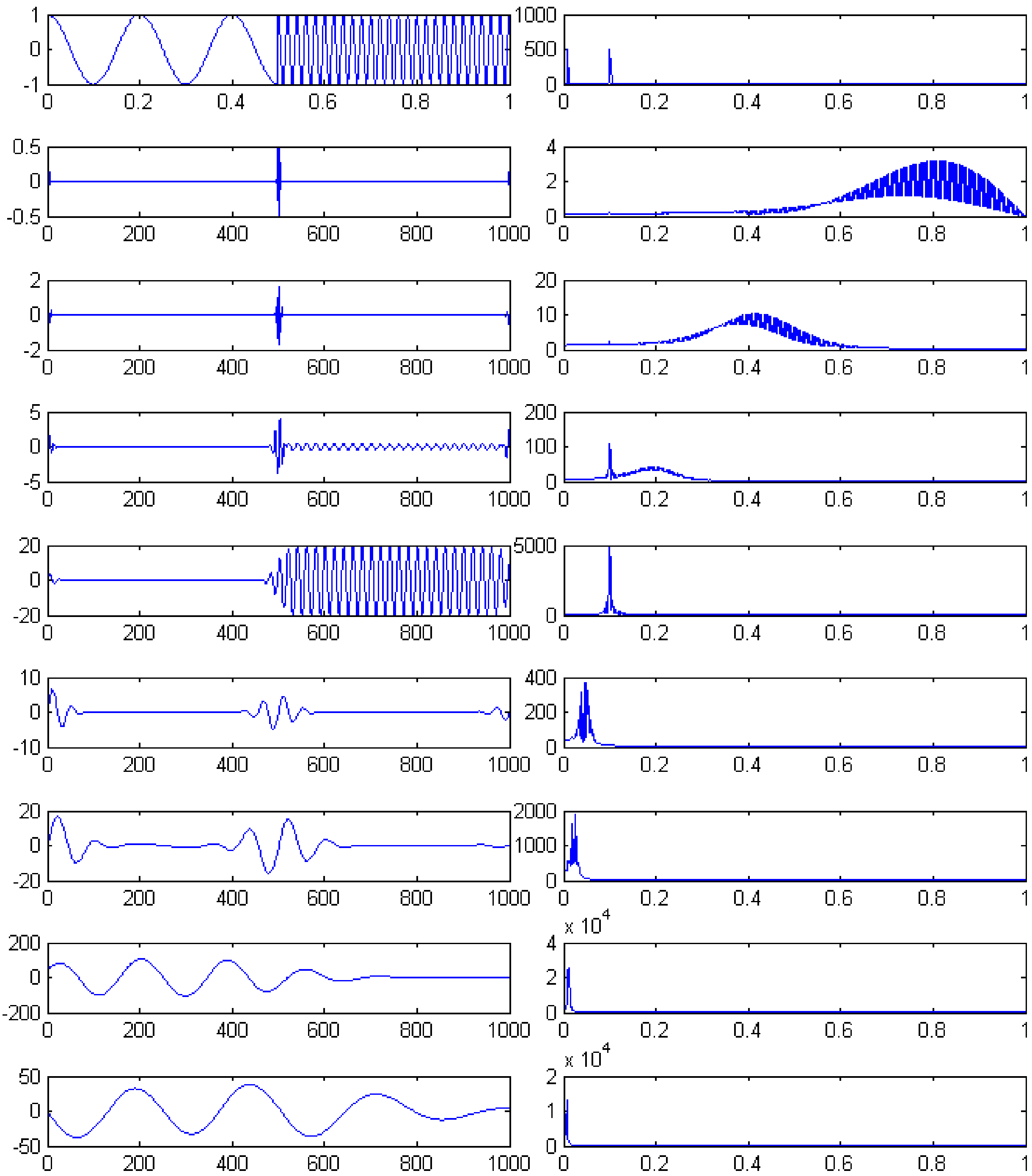


*Fig.2.* Wavelet transform of the function (1). The top right plot is the signal, the top left plot - the Fourier transform of the signal. Below, on the right side are the details ( $a = 2, 4, 8, 16, 32, 64, 128, 256$ ) of the wavelet transform (Morlet wavelet) of the signal, on the left side - frequency respond (Fourier transform).

Source: designed by authors.

As we see on fig.1 and fig.2 the localization of the frequency of the function presented only with wavelet transform.

**Continuous wavelet transform**

Before continuous wavelet transform calculation must be done the input signal segmentation [2, 3, 4]. Than, for the spectrogram's values calculation must be set some parameters (fig. 3), as: $\omega_{\min}$ - minimum frequency value (boundary from above); $\omega_{\max}$ - maximum frequency value (boundary from below); $\Delta_\omega = \dfrac{\omega_{\max} - \omega_{\min}}{N_\omega}$ - the step by frequency (vertical axis), and $N_\omega + 1$ - the number of frequencies; $t_{\min}$ - the minimum value of the time of the selected fragment with the QRS-complex (left border); $t_{\max}$ - the maximum value of the time of the selected fragment with the QRS-complex (right border); $\Delta_t = \dfrac{t_{\max} - t_{\min}}{N_t}$ - the step by time (horizontal axis), and $N_t + 1$ - the number of moments by time.

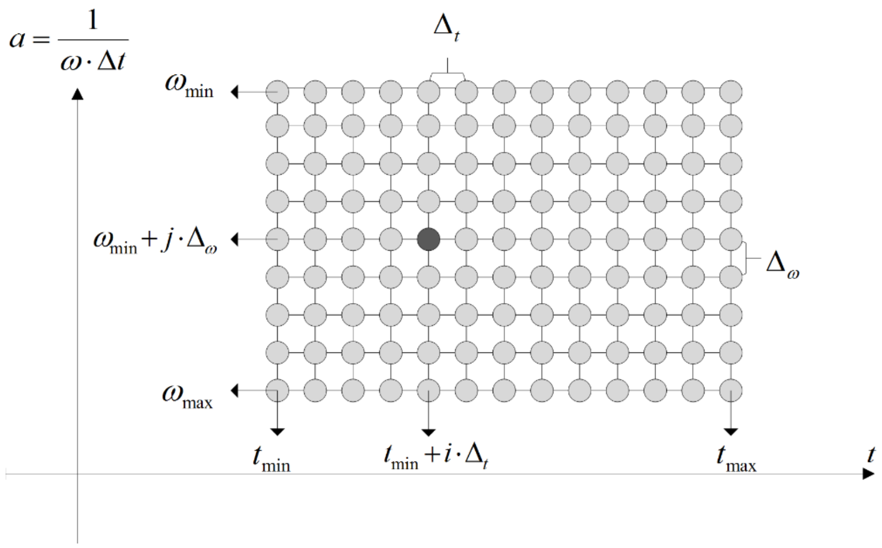


*Fig.3.* Spectrogram grid.

Source: designed by authors.

The time boundaries calculate on the stage of signal segmentation. The grid step by time for simplicity can be took equal to the original discretization frequency of the signal.

The boundary values of the frequencies and grid step by frequency can be calculated by the original signal's discretization frequency and the length of the selected segment. The maximal value of the frequency in accordance with the Kotel'nikov-Shannon-Neuquist theorem must be lower or equal to Nyquist frequency (2) [5].

$$\omega_{\max} \le \omega_{Nyqist} = \frac{1}{2} \cdot \omega \qquad (2),$$

where $\omega$ - the original discretization frequency of the signal; $\omega_{Nyqist}$ - Nyquist frequency.

The minimum frequency efficient to study with wavelet analysis is determined by the signal (segment) length. It is reasonable to make a convolution of signal with the wavelet with the carrier not less than the signal (segment) length (3) [6]:

$$\omega_{\min} \ge \frac{2}{T} \qquad (3),$$

where $T$ is the length of signal (segment).

$$C(a,b) = \frac{1}{\sqrt{|a|}} \cdot \int_{-\infty}^{+\infty} f(t) \cdot \psi^{*}\left(\frac{t-b}{a}\right) dt \qquad (4),$$

where $a, b \in R, a \ne 0$ - wavelet parameters determined scale and shift of wavelet function; $f(t)$ - input signal, $\psi$ - wavelet function and $\psi^{*}$ - scaling function.

If $a = \frac{1}{2^j}$ and $b = \frac{k}{2^j}$ then the formula (4) takes the form (5), that is, the discrete wavelet transform (DWT), the calculation of the coefficients

Wavelet spectrograms of which can be performed according to Mallat algorithm [7].

$$C[a,b] = C\left[\frac{1}{2^j}, \frac{k}{2^j}\right] = \sqrt{2^j} \cdot \int_{-\infty}^{+\infty} f(t) \cdot \psi^{*}\left(t \cdot 2^j - k\right) dt \qquad (5),$$

Where $j \in Z$ is the order of the wavelet decomposition; $k \in Z$ is the shift by an input signal.

To the frequency value calculation for the particular value of the parameter $a \in R, a \ne 0$ - dimensionless scaling parameter (not in hertz) - can be used formula (6).

$$a = \frac{\omega}{\omega_i} \qquad (6),$$

where $\omega_i$ - frequency of wavelet (wavelet decomposition); $\omega$ - discretization frequency of signal

As we see $a$ sets the scale of the wavelet function in frequency is equal to the ratio of the discretization frequency of the original signal to the wavelet (wavelet decomposition) frequency (fig. 4).

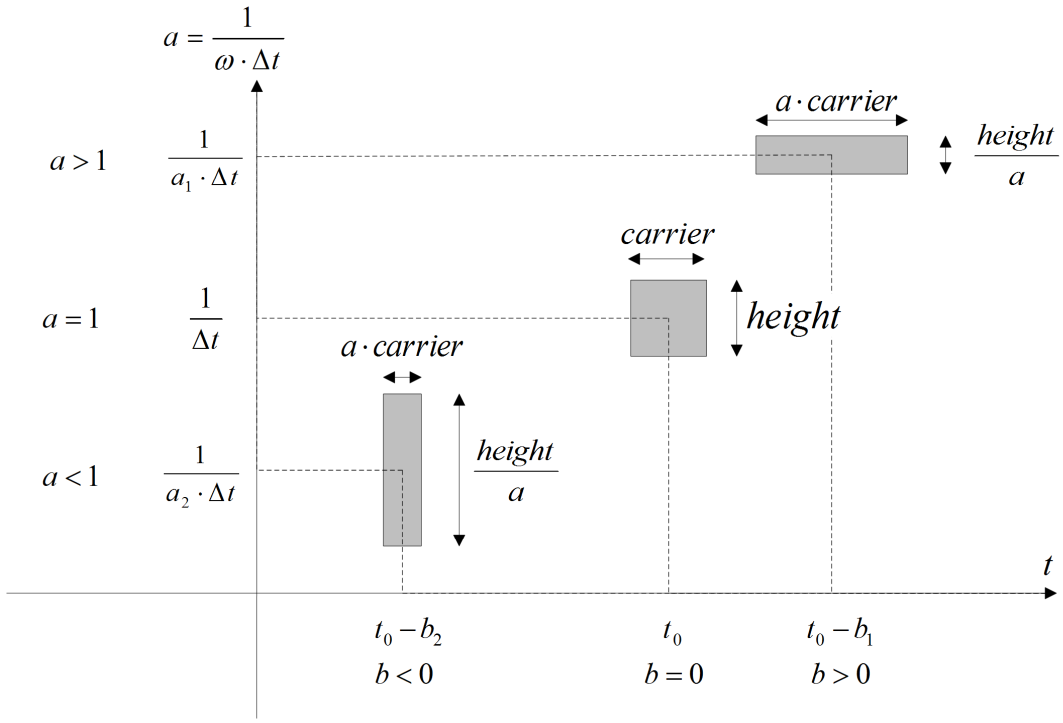


*Fig. 4.* Time-frequency resolution of wavelet

Source: designed by authors.

The value of parameter $b$ sets the time shift. In this case, the minimal and maximal values of the shift $b$ must be specified taking into account the length of the wavelet carrier (7), since otherwise time localization will be disrupted, and the wavelet spectrogram graph will have a "slope" to the right.

$$\psi^*\left(\frac{t-b}{a}\right) \Rightarrow \psi^*\left(\frac{t-\left(b-\frac{carrier \cdot a}{2}\right)}{a}\right) \qquad (7),$$

where $carrier$ - the length of the carrier of the wavelet (for wavelets with infinite carriers - the length of the significant carrier of the wavelet).

The final formula for spectrogram calculation at the points of the grid (continuous wavelet transform) will be (8):

$$C[a,b] = \frac{1}{\sqrt{|a|}} \cdot \int_{-\frac{carrier}{2}}^{+\frac{carrier}{2}} f(t) \cdot \psi^* \left( \frac{t - \left( b - \frac{carrier \cdot a}{2} \right)}{a} \right) \cdot dt \qquad (8)$$

where $+\frac{carrier}{2}$ and $-\frac{carrier}{2}$ - the right and the left boundaries of the wavelet.

For continuous wavelet transform by formula (8) any wavelet function can be used. Among the criteria for choosing a wavelet function, we can distinguish [8]:

- the number of zero moments of the wavelet, which is defined as the maximum value of $k$, at which the integral (9) vanishes (the higher the zero moment of the wavelet, the more complex the shape of it and, as a result, more accurately presentation of the complicated signals; the higher the zero moment of the wavelet, the bigger its carrier; the polynomial of the $k$-th order will not be detected by the wavelet):

$$\int_{-\infty}^{+\infty} x^k \cdot \psi(x) dx = 0 \qquad (9);$$

- the degree of smoothness;
- Fourier image;
- Compactness of the carrier;
- Symmetry;
- Orthogonality;
- Complexity.

Despite the existence of many criteria, the problem of choosing the wavelet for concrete problem is still a fundamental problem. In practice, there is no definite criteria. Most often, such criteria is to obtain better results using one of the wavelet function [9].

## Conclusions

Thus, in this article presented comparative analysis of Fourier and wavelet transforms. There was presented some general statements of the spectrogram gird calculation for preprocessing the signal for further classification problem solving.

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