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# *MODERN CONSTRUCTIVISM AND THE ONTOLOGICAL FOUNDATIONS OF MATHEMATICS*

*SUMMARY. This article is devoted to the clarification of modern constructivist methodology's role in research into the ontological foundations of mathematics. The possibility of overcoming a tendency in constructivism according to which mathematical knowledge serves the organizing of the subject's inner world, instead of tasks of objective ontological reality description, is shown.*

*KEY WORDS. Ontological foundations, objective reality, mathematical object, construction.*

Constructivism as one of the influential philosophical trends of the past is undergoing a rebirth today and is obtaining new peculiarities. First of all, there is an attempt to take into account special sciences including human science — this is how in philosophical discourse the notions of radical, communicative, social, methodological, utopian, cultural, etc. constructivism have appeared. However, the genetic connection of new constructivist trends with such a specific field of knowledge as mathematics is often beyond consideration. In this article, we shall try not only to demonstrate this connection, but to show what role the quickly developing methodology of modern constructivism can play in research into the existence of the particular kind abstract subjects that the objects of mathematics constitute.

Let us address the background of the problem. It is doubtless that Kant should be considered the first philosopher-constructivist to address the world of experience, perceived by empirical consciousness as really existing, as a construction, as the result of the transcendental subject's activity. Actually, consistence reasoning and a widespread constructivist understanding of consciousness are present in the works of the great German thinker (despite the fact that "constructions of reason" as the foundation of learning were actually considered not only by Kant, but for example by Lambert who wrote to him). It is notable that attention is not always concentrated upon the character of constructing itself according to Kant, so in general it is possible to talk about Kant's constructiveness of: a) a priori mathematical objects; b) primary principles of natural science; c) metaphysical ideas. That is why it is not surprising that Kant's constructivism is called epistemological, not mathematical.

On the other hand, the *idea* of constructivism has a long history in European philosophy. Concerning that, E.L. Chertkova rightly remarks: "As a theoretical idea, constructivism was practiced in ancient times, especially in works of antique mathematicians, for example Eudoxus of Cnidus, Plato's junior contemporary and

opponent, who proved the constructivist origin of mathematical objects" [1; 118]. Kant supports Eudoxus' point of view according to which "as evidence of a mathematical object's existence there is pointing out the principles of its construction or the opportunity of its analysis as a definite construction" [2; 66]. I.T. Kasavin writes: "In a narrow sense, constructiveness related to Kant just concerns Kant's understanding of mathematics. Kant uses the notion of constructiveness to separate philosophy from mathematics" [2; 66]. A.V. Kozein notes: "Unlike Lambert, Kant doubts the opportunity of putting constructivist experience into philosophy. This leads to outlining mathematics as a constructivist science" [3; 8]. Thus, it is no wonder that mathematical constructivism became the first influential constructivist concept in epistemology at the end of the 19th Century, i.e. constructivism in mathematics' justification (early intuitionists were L. Kronecker, E. Borel, A. Poincaré). Beginning with 1907, constructivism underwent considerable development in the form of the intuition concept of the Dutch scientist L.E.J. Brouwer, who understood by mathematics a science about intuitionally evident mental constructions free of the "dictate" of logic and language. Intuitionist research did not stop at that — they are continued by a number of outstanding scientists among whom there should be marked A.A. Markov, E. Bishop, D. Van Dalen, A.S. Troelstra.

Rapidly developing, within one or two decades, the constructivist trend has survived a serious "subject slide". Now the first place is occupied not by the problem of knowledge foundation but the process of reality constructing, a part of which is the watching, learning and creating subject. The similar change in "research orientation" attracts scientists working in different branches of science and technology: biologists, neuro-cyberneticists, psychologists, sociologists, etc. to the ranks of constructivists. The problems of the leading, defining "mathematical" orientation of the constructive movement recede. The most famous trends resulting from this are *radical constructivism* and *social constructionism*. Therefore, due to a number of recognized weak points in classical mathematical constructivism in its attempts to build an integral system of mathematics foundations, there arises a question: can new forms of constructivism enrich, expand or direct the methodology of mathematical object existence problems research not in a special-scientific but in a *philosophical* sense? To answer this question, let us examine the types of modern constructivism described above.

*Radical epistemological constructivism* is a concept relying on the theory of autopoetic (from the Greek "autos" — self; and "poien" — to create) systems developed by Chilean biologists F. Varela and H. Maturana (who, in turn, inherited the ideas of S. Ceccato). Austrian (P. Vatslavik, H. von Foerster, E. von Glasersfeld) and German (G. Roth, etc.) scientists are identified as the founders and main representatives of this concept. According to it, the experienced world is nothing but the product of our brain activity — there is only reality constructed by us and that we are responsible for. With that, as A.V. Kozein delicately notices, "to understand the specifics of radical constructivism it is important to differentiate between two interrelated, but different levels: firstly the level of causal-biological reconstruction of sensitive experience (cognition), secondly the meta-level of cognition theory, at which definite philosophical conclusions made on the basis of biological reconstructions develop" [3; 3]. Further on we shall work with this

second *meta-level*, but with one remark: philosophical conclusions, obtained this way, can be articulated not only gnoseologically but ontologically, axiologically, etc. It is more important for us, as our research concerns this very profound, existential layer of the mathematical object's appearance and functioning. So in this respect, the most important philosophical statement of radical constructivism can be formulated as the following thesis: "Cognition serves to organize the subject's inner world but not the task of objective ontological reality description" [3; 15]. The question of independence of objective reality from the system of notions, cognitive schemes, etc. is considered by the representatives of this trend as senseless. In our opinion, it can be partially explained by the fact that the answer to it can't be found. E.Y. Rezhabek, for example, comments upon Maturana and Varela's statement on the exceptionally "inner", individual determination of neuron activity allowing every separate person to see colour the following way: "But let's ask the following question: how has the Sun's light changed after leaving creatures with apparatus of colour vision? ... Has the sunlight color spectrum changed after radical structural growth of living creatures on the evolutionary ladder? No, it has evidently not" [4; 72].

Thus, according to V.A. Lectorsky's remark, "a strange situation" is observed: "it is possible to speak about autopoetic systems only on the condition of understanding these systems as really existing in the real environment and in cooperation with it. It turns out that "the world is in the mind and the mind is in the world". I even don't say that it is impossible to understand inner changes in systems of such kind, if the necessity of getting information from the outer world is not considered" [5; 35]. We suppose that the problem of *objective reality*'s independence on whatever it might be that is discussed in radical constructivism up to its most extreme form, epistemic solipsism (E. von Glasersfeld) will actually be senseless until, speaking the language of mathematics, "the function" and "the argument" are swapped. Why not put the question of *cognition schemes and constructs of consciousness* dependence in terms of the structures of existence, not vice versa? From the point of view of ontology understood as "the science of categorical structure of any subject" [6; 16], it would be quite logical. Nevertheless it is highly possible that it would not satisfy the followers of this concept, because of an evident contradiction with its main principles.

In any case, the situation is really "strange", as proved by statements by constructivists themselves. Particularly according to S. Schmidt, a specialist in neuro-dynamics, the radical constructivist does not deny *reality*, though announces his position in the following way: "... all statements on reality are a hundred percent my experience" [7; 35]. In a number of works, famous neurophysiologist and constructivist G. Roth also states an idea that in biology any constructions become useless if not to accept the existence of a consciousness-independent world with its order allowing life itself [8]. It might seem strange, but radical constructivists *actually* do not refuse the existence of the outer world independent of them until imagination of it is outside of everyday life boundaries ("inner realism" so to say).

*Social constructivism (constructionism)* differs from other types of constructivism at least by the fact that it was formed within the frames of social-human knowledge. It is considered that its first historical version is the externalist "strong program" of the Edinburgh School (D. Bloor, B. Barnes, S. Shapin,

E. Pickering, etc.). Modern literature review allows us to draw a conclusion on the considerable role of ideas of internalist trend representatives (B. Latour, S. Woolgar, etc.).

In this trend, the practice of obtaining knowledge by means of constructing social experience but not objective reality experienced by the object is studied. The stress is on the activity of the mental world, as E.L. Chertkova remarks: "this stress is in tune with its epoch: the more man thinks that he is getting free of nature, the more importance he gives to the mental world created by him" [1; 123]. Meaning does not represent any reality, forming this reality *itself*. The fight with fundamentalism of any kind brings social constructivists to reject ontological problems: "Cognition serves to order the inner world of the social subject, but not to explain the objective ontology of existence" [1; 124]. Thus moving away from the "sociology" of constructivism with "actor nets", STS settings ("science and technology research"), "hybrid objects", etc., coming back to the level of philological reflexion, we get the same thesis as for radical constructivism: "Cognition serves to organize the inner world of the subject but not to fulfill the tasks of objective ontological reality description". From this it follows that criticism of social constructivism settings on the grounds of ontology is the same: "As for social constructivism, it cannot reduce all real processes to a construction. Because it has to start with the fact that social processes constructing cognition, knowledge and the world of subjectivity really exist. People having active-communicative relations really exist. Subjects created by people in which social and cultural senses are objectivized exist" [5; 36]. The situation paradoxes are aggravated by the fact that constructivists, giving subjects the privilege to construct reality, find an alternative to the "naked" object in the form of the "authentic", taken outside of social events and factors. In other words, we come back to the classical dichotomy scheme "phenomenon-reality" that indicates the ambitious demands of the "new" philosophy in its attempts to break with traditional settings. I. Hacking remarks: "Though social constructivists are warming up in the rays of the Sun that they call post-modernism, actually they are rather old-fashioned" [9; 49].

Together with this, social constructivism, rather than radical, turned out to be a much more "movable", flexible, open-to-dialogue phenomenon. According to O.E. Stolyarova, "tight constructivist shoes start to hurt" the constructivists of the end of the 20$^{th}$ Century. It is possible that many researchers come to the conclusion of the impossibility of further development of the scientific theory against the background of evident contradictions and paradoxes in the philosophical foundations of the given trend. There is a necessity in building such theoretical models that might admit the ontological compatibility of reality and the construction, not refusing the existence of either of them. Cognitive activity is *really* included in social-cultural reality, the activity of consciousness is *really* constructive, they *actually* can't help having features of sociality but together with this they are not obliged to be driven to the last point! It is remarkable that B. Latour and S. Woolgar in the second edition of their famous monograph "The Life of the laboratory: social construction of scientific facts" delete the term "social" from the title. A.N. Whitehead contributed to the development of social constructivism ideas in the stated trend, he "managed to give the construction universally ontological importance with which it does not contradict reality but points at it" [10; 98].

Let us suppose the presence of an ontological setting common to all types of constructivism and try to designate the modern outlines of *mathematical* constructivism as one of the trends in mathematics philosophy of the end of the 20th and the beginning of the 21st centuries. In the brief excursion on its history presented above we mentioned the research of L.E.J. Brouwer, and critical remarks developed in works by outstanding domestic and foreign scientists.

Thus, beginning in the 1940s, partially based on the works of A.N. Kolmogorov, the Soviet School of constructive mathematics forms (A.A. Markov, N.A. Shanin, A.G. Dragalin and their students). The main "philosophical" differences of the Soviet constructivism from both intuitionist mathematics and other constructive trends are: a) refusal of Brauer's idea of the subjective beginning in mathematics; b) the key role of the artificial language by means of which basic constructive objects are given. Regardless of the fact that the official ideology of the School is natural scientific material, the ontological status of the mathematical object is rather defined nominally here. Approximately at this time, the American mathematician E. Bishop developed his original version of constructivism [11]. Different from the Soviet School in a number of mathematical principles as well as in understanding the number as a primary object of mathematics given the subjective reality in Kant's style, his constructive mathematics tends to break free from "philosophical dogma related to its objects" on the whole [12; 88]. Similarly, modern followers of Brouwer's concept (D. Van Dalen, A.S. Troelstra) and version developers of predicative (S. Feferman, H. Freedman, K. Schütte), methodological (G. Dingler and his followers) and liberal (P. Martin-Löf) constructivism and researchers into the possibilities of computer-constructed mathematical objects (T. Timoshko, partially R. Hersh and others) appear, firstly, as working *mathematicians, logistics specialists, computer programmers*. The programs of most of them became famous more in mathematical than in philosophical sciences, presenting some mathematical constructions potentially interesting for "pure" philosophical reflexion.

On the other hand, some researchers point out a *social-constructivist* approach in the philosophy of mathematics, according to which mathematics is the product of social activity and culture on the whole (T. Timoshko, R. Hersh, P. Ernest) [13]. Within the frames of the given approach mathematics can be called "humanistic", i.e. such science that could be considered as the human activity, a social phenomenon, a part of the human culture from the philosophical point of view. R Hersh writes: "Customs, traditions and institutions of our society are real despite the fact that they can be related neither to the subjective nor to the outer-human world. It's another social-cultural-historical reality. Mathematics is just this third type of reality — the "inner" one — in relation to society in general, and the outer one in relation to each of us particularly" [4; 16-17]. V.V. Tselistchev evaluates the given position the following way: "The matter is that admitting mathematics as just a human activity, from a humanistic mathematics point of view, has no bearing on the philosophy of mathematics at all. Mathematics sees a hidden sense behind the frames of the social-historical-cultural context which is revealed in the unchangeable ontology of mathematical subjects and in the out-of-time character of mathematical truths. But if, as humanistic mathematics proves, mathematical cognition is not exact, then the truth and ontology in mathematics change during the process of cognition" [15; 139]. Considering the

criticism of the famous domestic philosopher of mathematics as categorical, we, nevertheless, agree with him in that principle moment that, like in the case with "classical" social constructivism, the group subject grades a number of world laws here, replacing the objective reality by itself.

In any case, the history of mathematical constructivism's development demonstrates that it can be dually considered: on the one hand, as a totality of mathematical programs, allowing to construct new mathematical objects and to produce exact calculative procedures on their basis; on the other hand, as a trend in the philosophy of mathematics having a definite reflexive tradition and development prospects. Concerning the second aspect, we'd like to remark that the ontological setting of mathematical constructivists does not differ from the common characteristics of all constructivist concepts, considered above. Objects of mathematics are the same mental constructions that structure reasoning and organize the subject's inner world despite the objective regularity of ontological reality. Similar to the cases mentioned above, the given position reveals its limitation and contradiction. In intuitionism, for example, the problem of *proper* foundations of intuition remains unclear, which was noticed by A. Heiting:" The notion of intuitive clarity in mathematics is not intuitively clear itself" [16; 225]. It should be underlined that we don't talk about the methodological difficulties that constructivists have faced in their attempt to reason mathematics (unsuccessful critiques of the law of the third excluded, principally constructively unprovable theorems, etc.). An unsettled problem is said to remain concerning the participation of the *world itself* in the process of research and description of its separate parts. It is remarkable that like in the case with radical and social types of constructivism, the practice of mathematical objects construction, beginning with L.E.Y. Brouwer, demands that we deal not only with the subjective but with objective reality as well. Thus one of the domestic researchers of intuitionist mathematics, A.S. Levchenko, comes to the conclusion that "in its foundation it supposes objective mathematical (logical) truths and objects, explanation of which in intuitionism reflects their theoretical, intuitive constituent" [7; 148].

In our opinion the real obstacle in the path of constructivism's development, including mathematical constructivism, was and is the problem of a *philosophical* view of "non-philosophical" things and phenomena. Besides, the necessity to differentiate between philosophical reflexion and the reflexion of practicing mathematicians makes us sure of the fact that the proximity and close interconnection between "the problem of mathematics reasoning" (in its classical version) and an absolutely ontological problem of the mathematical object's existence does not give a right yet to assimilate between these subject fields. We also suppose that the general problem of mathematical *reasoning* can and must be philosophically articulated, in the extreme case motivating the researcher to search for an answer to the question: does the mathematical object *exist*? Among other approaches the constructive approach is to play an important role here.

Let us draw some conclusions. In this article we tried to analyze the influence of the general trend of one of the most powerful methodologies, called constructivism, upon the research of the peculiar, "out-of-experience" branch of reality represented by mathematics and its objects. Despite the critical position we take with regard to the given trend as the whole, a number of positive points, that give an optimistic

view of the methodological opportunities of modern mathematical constructivism, should be marked.

Firstly, despite the contents of these or those constructivist concepts, it can't help being noticed, that all of them, one way or another, attract the attention to fundamental ontological and gnoseological problems, making dogmatized "truths" impeccability doubtful. It is especially relevant for the philosophy of mathematics, which managed to accumulate a serious list of "theatre ghosts" during the history of its development and, as a consequence, "eternal questions" still without answers.

Secondly, the revealing of contradictions in the ontological background of the constructivist trend brings heuristic value, as it stimulates searches for new research on referent points instead of the lost ones, which is important in clearing up such complicated and hard to explain phenomena as, for example, crises and revolutions in mathematics.

Thirdly, the appeal to mental structures of the mind and psycho-physiological states of the subject, as immanent characteristics of experienced activity, opens to constructivism the prospect of a methodological particularization quite relevant actual in conditions of non-classical rationality's "triumph", refusal of atomistic ontology and "naпve" objectivism, allotting things with "human-size properties", etc. Even with the account of such non-specific "non-humanitarian" objects, like mathematical ones, present as ontologically invariant structures, it shouldn't be forgotten that these objects are created, researched and transmitted by *man*, including into this or that social tradition with a necessity.

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