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THE PROBLEM OF THE PRESSURE RECOVERY OF A FORMATION FLUID

ABSTRACT. A considerable part of natural oil and gas formations are of fractured or fractured porous type. These formations are made up of non-porous or porous block matrices separated by a system of interconnected fractures. Such macroscopic properties of porous blocks and fractures as porosity and permeability differ radically. To analyze the filtration process in fractured porous formations, the linear approaches are usually used, among which is a popular linear model, based on the dual porosity concept. Meanwhile, numerous field observations indicate high non-linear behavior of oil and gas fractured formations under their development. In this case linear models cannot be used to analyze the filtration process. To determine or update macroscopic parameters of formations in oil field practice, pressure recovery processes are often used. Simulation results of pressure recovery in linear formations for Newtonian and non-Newtonian fluids can be found in publications. In the present article a non-linear option of the double porosity concept is used. It is assumed that effective formation porosity and permeability depend heavily both on effective stress tensor and fluid pressure. Fracture porosity decreases when fluid pressure in fractures drops. This approach reflects main features of filtration processes in natural fractured formations. The elastic-plastic mode of the pressure recovery in a non-Newtonian fluid in a deformable fractured porous formation is considered. Field data interpretation is different as compared to linear models. Consequently, the obtained estimate of oil reserves can also differ.

KEY WORDS. Pressure recovery, elastic-plastic mode.

Introduction

Many natural oil and gas fields have fractured structure [1–6]. In practice, linear model of a fractured medium is widely spread [7]. But many fisheries observations suggest non-linear character of filtration processes in such reservoirs [8–10]. Effective reservoir characteristics can vary greatly, which can cause errors in estimating reserves of raw materials. To determine the effective characteristics, various methods including pressure transient analysis are used. The well is closed for a few hours, and a pressure recovery curve is registered. Using a specific reservoir model, it is possible to determine the parameters of the curve formation.

In this paper, the problem of pressure recovery is analyzed within the scope of non-linear approach [11], which well corresponds with real media [12, 13]. The fluid with a limiting gradient is taken as a fluid model [14] reflecting the behavior of real oil—the flow starts when the pressure drop exceeds a certain threshold. The elastic-plastic mode of pressure recovery is considered for the first time in such perspective. The elastic-plastic mode was introduced for linear Newtonian fluid mechanics in [7]. Pressure recovery in the linear visco-plastic fluid reservoir is considered in [15].

Statement

Next, it is considered filtering of the well (cylindrical symmetry), but a similar analysis was performed for the case of parallel flow to the gallery operating with a constant bottom hole pressure until closing-in.

Under the approach [11], elastic-plastic fluid pressure recovery mode with limiting gradient in fractured porous medium is described by the equations in dimensionless form:

$$\frac{\partial\phi}{\partial\omega}=\kappa_1\frac{1}{r}\frac{\partial}{\partial r}\left[r\phi^n\left(\frac{\partial\phi}{\partial r}-H\right)\right]-\alpha\phi,\ \frac{\partial\phi}{\partial\omega}<0, \tag{1}$$

$$\frac{\partial\phi}{\partial\omega}=\kappa_2\frac{1}{r}\frac{\partial}{\partial r}\left[r\phi^n\left(\frac{\partial\phi}{\partial r}-H\right)\right]-\alpha\phi,\ \frac{\partial\phi}{\partial\omega}>0. \tag{2}$$

In (1), (2), $0<\phi(r,\omega)\leq 1$ —the fluid pressure in the main ducts (cracks), r, ω — a coordinate and time, H — a limiting gradient, κ_i — piezoconductivity coefficient. Parameter n takes on the value of 0 and 3 for a linear [7] and a non-linear model [11], respectively.

The equation (1) describes the selection of filtration liquid, the equation (2)—the restoration of pressure in the reservoir after the termination criteria. In accordance with [11] the equations are meaningful only if $\phi>0$ when the dimension pressure in the cracks exceeds a critical value. When $\phi\leq 0$ cracks are closed and in this zone filtration is carried out only in a porous matrix. Member $-\alpha\phi$ describes the fluid flow from the fracture into the porous matrix. This approach is justified because when removing the pressure transient, the well is closed for a short enough period of time (several hours) and the influence of the porous matrix does not have time to fully manifest. In case of purely fractured reservoir (non-porous matrix) – $\alpha=0$. The filtering process in fractured porous medium is described in full by a system of two equations for the pressure in the fractures and the porous matrix. When elastic recovery mode pressure in (1), (2) is $\kappa_1=\kappa_2$. However, in real environments discharge formation is 'tougher' and (elastic-plastic model [7]).

Assume that the well had been working with a constant flow rate of q up to the time $\omega=\Omega$. At the time Ω the well closes (flow rate of 0) and the pressure is restored.

The initial and boundary conditions for the equations (1), (2) (n = 3) have the form:

$$\phi(r,0)=1 \tag{3}$$

$$r\phi^3\left(\frac{\partial\phi}{\partial r}-H\right)=q,\ r=r_0,\ 0<\omega<\Omega, \tag{4}$$

$$r\phi^3\left(\frac{\partial\phi}{\partial r}-H\right)=0,\ r=r_0,\ \omega\geq\Omega, \tag{5}$$

where $r_0 << 1$ — well radius.

After the well is closed, the discharge wave front [7] $r = \mu(\omega)$ in the reservoir is formed where the derivative is $\frac{\partial\phi}{\partial\omega}(\mu(\omega),\omega)=0$, breaking the reservoir into two areas. In the area adjacent to the wellbore pressure increases and, consequently, holds the equation (2). In the outer area the pressure continues to drop and holds the equation (1). With increasing time the discharge wave front moves from the well along the reservoir.

Solution of the problem

Solution of the problem (1)—(5) is constructed by the integral method [7].

When the well works, the pressure distribution in the reservoir is sought in an approximate form:

$$\phi=\phi_1=\tilde{N}ln\left(\frac{r}{l}\right)+A_0+A_1\left(\frac{r}{l}\right)+A_2\left(\frac{r}{l}\right)^2 \tag{6}$$

In (6), $l = l(\omega)$ — a dummy border, such one that the pressure distribution inside $r_0 \leq r < l(\omega)$ area is affected by the working well and outside the area it is unperturbed.

The coefficients $\tilde{N}$, A_i are determined by the boundary conditions at the well (4) and the conditions at the dummy border of bore hole effect $l = l(\omega)$:

$$\phi=1,\ r\phi^3\left(\frac{\partial\phi}{\partial r}-H\right)=0,\ \frac{\partial^2\phi}{\partial r^2}=0,\ r=l(\omega). \tag{7}$$

Using (7), we will calculate coefficients A_i with the help of C:

$$\grave{A}_0=1+\frac{3}{2}C-Hl,\ \grave{A}_1=Hl-2C,\ \grave{A}_2=\frac{1}{2}C. \tag{8}$$

The condition at the well (4) gives an algebraic equation, implicitly defining C as a function $l(\omega)$ which after discarding the small terms of order on r_0 has the form:

$$C^4\left(\frac{3}{2}+ln\left(\frac{r_0}{l}\right)\right)+C^3(1-Hl)=q^{\frac{1}{3}}. \tag{9}$$

Integrating the equation (1) multiplied by r in the perturbed region $0 \leq r \leq l(\omega)$ results in the differential equation:

$$\left(Cl+6Hl^2+\frac{1}{2}\frac{\partial C}{\partial l}l^2\right)\frac{dl}{d\omega}=12\kappa_1 q+\frac{1}{2}\alpha l^2(12-C-4Hl). \tag{10}$$

The derivative $\frac{\partial \tilde{N}}{\partial l}$ in (10) is calculated by the equation:

$$\frac{\partial \tilde{N}}{\partial l}=\frac{C^2+CHl}{4Cl\left(ln\left(\frac{r_0}{l}\right)+\frac{3}{2}\right)+3l(1-Hl)}. \tag{11}$$

Differential equation (10) is solved numerically using (9), (11) and the initial condition $l(0) = 0$. After determining the law of motion of the boundary $l = l(\omega)$ and the dependence of the coefficient $C = C(l)$, the pressure profile is defined by (6), (8).

Distribution of pressure after closing-in of the well will be calculated as:

$$\phi(r,\omega)=\begin{cases}\phi_2, & 0\le r\le \lambda(\omega),\\ \phi_1, & \lambda(\omega)<r<l(\omega),\\ 1, & r\ge l(\omega),\end{cases} \tag{12}$$

where $\lambda = \lambda(\omega)$— new disturbance front induced by closing-in of the well

$$\phi_2=\hat{A}_0+B_1\left(\frac{r}{\lambda}\right)+B_2\left(\frac{r}{\lambda}\right)^2. \tag{13}$$

Coefficients B_i are determined by the condition of continuity and smoothness of the pressure on the perturbation front:

$$\phi_2=\phi_1,\ \frac{\partial\phi_2}{\partial r}=\frac{\partial\phi_1}{\partial r},\ \frac{\partial^2\phi_2}{\partial r^2}=\frac{\partial^2\phi_1}{\partial r^2},\ r=\lambda(\omega).$$

We find

$$\mathrm{B}_0=1+\tilde{N}ln(z)-Hl,\ \mathrm{B}_1=2\tilde{N}(1-z)+H\lambda,\ \mathrm{B}_2=-\frac{1}{2}\tilde{N}(1-z^2), \tag{14}$$

where

$$z=z(\omega)=\frac{\lambda(\omega)}{l(\omega)}. \tag{15}$$

Integration of the equations (1), (2) in the perturbed region results in a differential equation with respect to $z(\omega)$:

$$\begin{aligned}Cl^2z\frac{dz}{d\omega}&=Cl\frac{dl}{d\omega}(5z^2-8z^3+3z^4)+6Hl^2\frac{dl}{d\omega}z^2-\\&-\frac{1}{2}\frac{\partial C}{\partial l}l^2\frac{dl}{d\omega}z^2\left[12ln(z)+13-16z+3z^2\right]+\\&+12(\kappa_2-\kappa_1)C(1-z)\left(2y-(1+z)y^2\right)+12\kappa_1C(1-z)^2+2\alpha Hl^3(3z^2-2z^3)-\\&-\alpha l^2z^2\left[6+6Cln(z)+\frac{1}{2}C\left(13-16z+3z^2\right)\right],\end{aligned} \tag{16}$$

where C and $\frac{\partial \tilde{N}}{\partial l}$ are determined by (9) and (11),

$$y = y(\omega) = \frac{\mu(\omega)}{\lambda(\omega)}. \tag{17}$$

In the case of an elastic medium in (16) $\kappa_1 = \kappa_2$.
The condition on the discharge wave front

$$0 = \frac{\partial\phi}{\partial\omega} = k_1 \frac{1}{r}\frac{\partial}{\partial r}\left[r\phi^3 \left(\frac{\partial\phi}{\partial r} - H \right) \right] - \alpha\phi, \ \mu = \mu(\omega),$$

leads to an algebraic equation relating y and z:

$$2\kappa_1\tilde{N}(1-z)(1-(1+z)y) - \alpha l^2 z^2 y\left[1 + \tilde{N}\ln(z) - Hl(1-zy) + 2\tilde{N}(1-z)y - \frac{1}{2}\tilde{N}(1-z^2)y^2\right] = 0. \tag{18}$$

In the absence of flow through the pores ($\alpha = 0$), we find from (18):

$$y = \frac{1}{1+z}. \tag{19}$$

Thus, the two unknown functions $z = z(\omega)$ and $y = y(\omega)$ are defined by the system of differential equations (16) with respect to $z(\omega)$ with the initial condition $z(0) = 0$ and the algebraic equation (18).

The position of the dummy border $\lambda(\omega)$ and the discharge wave front $\mu(\omega)$ are determined by (15), (17):

$$\lambda(\omega) = l(\omega)z(\omega), \ \mu(\omega) = l(\omega)z(\omega)y(\omega).$$

The pressure distribution in the reservoir has the form of (12).

Conclusions

Based on the analysis, we can draw conclusions about the impact of fluid parameters (limiting gradient), medium (linear/non-linear, elastic/elastic-plastic, with a drain on the porous matrix and without flow) and the initial filter parameters (bottom hole pressure, flow rate) on the recovery pressure process.

With increasing initial gradient H, as well as increasing piezoconductivity coefficient $k_{1,}$ functions $l(\omega)$ and $\lambda(\omega)$ decrease (Fig. 1). Therefore, the size of the zone of mode influence in the well is the biggest for Newtonian fluid and decreases with rising H. Thus, the increase of visco-plastic properties of the fluid causes reduction in a propagation speed of perturbance in the reservoir and narrows a depression cone. With increasing H, function $z = \lambda/l$ grows. The propagation of a discharge wave $\mu(\omega)$ along the reservoir is slower with increasing limiting gradient because of a slower fluid filtration processes. After closing-in the well, pressure recovery is slower for large values of limiting gradient, but this is partly balanced by the distribution of the fluid pressure in reservoir before closing-in.

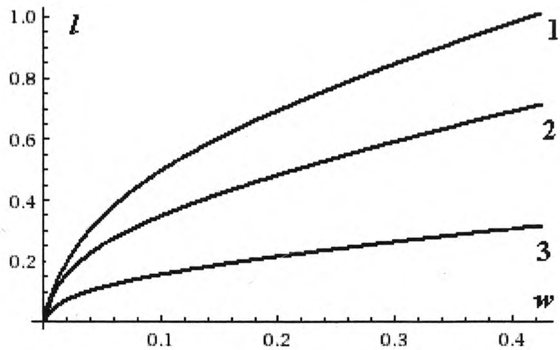

Fig. 1. Dummy border movement *l(w): k=0,01 (1), k=0,05 (2), k=0,1 (3)*

Recovery of pressure in the reservoir is shown in Fig. 2. Inflection point—the discharge wave front.

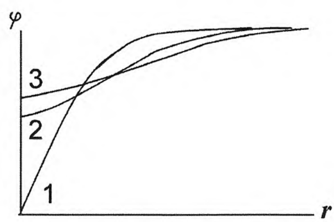

Fig. 2. Recovery of pressure in the reservoir at closing-in—(1), at regular intervals after closing-in—(2), (3)

In a linear medium, the speed of dummy boundaries $l(\omega)$, $\lambda(\omega)$ and discharge waves along the reservoir is higher than in a non-linear one. However, pressure is restored faster in a non-linear medium. With the growth of the initial gradient, the difference in the speed of dummy boundaries in linear and non-linear medium becomes less noticeable, but the difference in the propagation speed of discharge waves and the speed of pressure recovery increases.

When elastic-plastic filtration mode, the propagation speed of dummy boundaries $\lambda(\omega)$ and discharge waves $\mu(\omega)$ is higher than in case of the elastic mode. These rates increase with increasing ratio of κ_2/κ_1. In the elastic-plastic medium pressure is recovering faster than in the elastic one. With the growth limiting gradient, the differences between the modes are reduced.

Parameter α describes the fluid outlet through the porous matrix. The rise of parameter α causes low speed of the discharge wave propagation and pressure recovery. This can be explained by the fact that the fluid outlet causes additional pressure drop. The rise of limiting gradient causes less influence of the outlet on the propagation speed of dummy boundaries in the perturbed regions, discharge waves, and the speed of pressure recovery.

The results can be used in oil field production to define or specify the parameters of productive reservoirs.

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