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# *TEMPERATURE WAVES IN THE GROUND NEAR THE BASE OF THERMAL CONSTRUCTION*

*ABSTRACT. Numerical study of transient heat conduction in the ground near the pile is presented in this paper. The ground and the pile are exposed to temperature waves, i.e. seasonal variations of air temperature. A process of heat conduction in the ground is described by a two-dimentional axisymmetric equation of transient heat conduction without a source term. The CONDUCT algorithm is used for numerical solution of the problem. The «stationary periodic» mode is being achieved during 5 periods in the present problem. The heat exchange in the pile is more intensive near the surface than down in the ground. Penetration depth of the temperature waves decreases with the depth exponentially therefore their influence on temperature distribution is the biggest near the upper bound. Thermal inertia demonstrates hyperbolic properties of solution for heat conduction equation with time-periodic boundary conditions. It contributes to phase difference between temperature oscillations at different depths.*

*KEY WORDS. Conduction, temperature waves, solid body, air.*

## Introduction

The heat propagation over a solid body is defined by establishing temperature as the position and time function. To find this function, it is necessary to compose a transient heat conduction equation which differentially illustrates the energy conservation principle in a solid body. Derivation of this equation is given in [1]. Generally there is no solution in the form of progressing wave for the classic parabolic equation of heat conduction. But the equation type can be found in boundary conditions. Solution of the heat conduction equation with periodic boundary conditions in the semi-infinite domain has the properties of the hyperbolic equation and is found in [2], [3].

However temperature waves can be observed only in seasonal changes of the air temperature. Some processes taking place in machines such as engines, are characterized by periodically repetitive operation with following temperature changes in the environment and body of the engine that happen under a certain rule. Here one can see the temperature field change periodically. The graphs for various repetitive heat processes are given in [4; 90]. Watching temperature waves pass let us calculate the heat conduction coefficient of a substance. Anders Jonas Ångström was the first to apply this method described in [5; 137]

Reinforced concrete parameters are taken as thermal and physical characteristics of the pile, and those of the ground are the heat conduction coefficient, density and specific thermal capacity of dry sand. The heat conduction coefficient of reinforced concrete is bigger than that of sand thus the pile can be treated as thermal construction in relation to the ground.

Should there be analytical solution to this problem it would be awkward because heat conduction coefficient is a 3D function. Analytical solution to a 1D problem on temperature wave distribution within a limited space is found in article [6]. Application of numerical methods to solve this problem seems simpler and more effective. They are given as in [7], [8].

**Problem statement**

Let us look at a plot of sandy ground which exchanges heat with the environment under the Newton-Richmann law (Fig.1). The Earth temperature is daily and seasonally periodic so environment temperature $T_{air}$ in this problem changes under the periodic law (Fig. 2). The law under which the air temperature changes with the time is such that minimum temperature is -40°C on January 15 and maximum temperature is +25°C on July 15. Change period $P$ for $T_{air}$ is equal to 1 year. We take the ground layer deep enough for the lowest bound to have constant temperature $T_{gr}$. Some additional impact on the temperature field will come from the concrete pile (Fig. 1). The right bound is a symmetry line of the construction basement. The left bound should be far from the construction, so the heat flow across it is equal to zero. The initial distribution of temperature is constant all around the area and equal to $T_{gr}$.

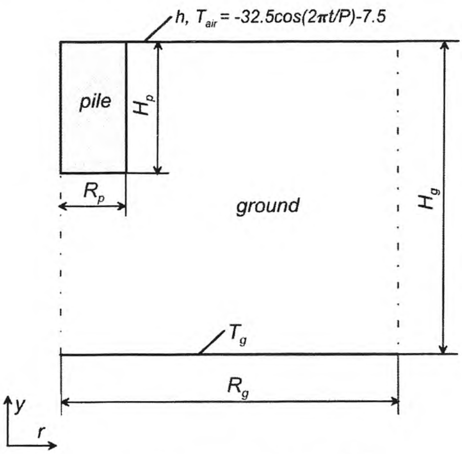


*Fig. 1.* The geometry of the problem

**Mathematic model**

The problem is solved by a 2D axisymmetric equation of transient heat conduction

$$\rho c\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)$$

$$\rho c=\begin{cases}\rho_p c_p,\ 0\le r\le R_p,\\ \quad H_g-H_p\le y\le H_g;\\ \rho_g c_g,\ \text{other}\end{cases}\qquad \kappa=\begin{cases}\kappa_p,\ 0\le r\le R_p,\\ \quad H_g-H_p\le y\le H_g.\\ \kappa_g,\ \text{other}\end{cases}\quad (4)$$

The initial condition has the following form

$$T|_{t=0}=T_g,\ 0\le r\le R_g,\ 0\le y\le H_g.$$

Boundary condition on the right bound is equivalent to zero of the first derivative of the temperature in the coordinate.

$$\left.\frac{\partial T}{\partial r}\right|_{r=R_g}=0,\ 0\le y\le H_g;$$

$$T|_{x=0}=T_g,\ 0\le r\le R_g;$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=H_g}=-\frac{h}{\kappa}\left(T|_{y=H_g}-T_{air}\right)0\le r\le R_g,$$

$$T_{air}=-7.5-32.5\cos(2\pi t/P).$$

**Calculating parameters and numerical scheme**

The algorithm CONDUCT [9] developed on the basis of the control volume approach [10] was used for the numerical solution of the problem. The computations were done on the proportional mesh 402 × 402 with 160,000 computational points, the pile having 400 of them.

Computational modeling was done for the following parameters:

1. Ground layer thickness $H_g$ and radius $R_g$ are 10 M.
2. Pile radius $R_p$ = 0.1 M, pile depth $H_p$ = 2 M.
3. Ground temperature $T_g$ = 2°C, which prevents its freezing.
4. The temperature wave impact shown in Fig. 2 is 2π/P, with period P = 31 536.000 being equivalent to 1 year in seconds.
5. The ground–air heat exchange coefficient is $h$ = 5 Wt/(m².°C).
6. The ground thermal and physical characteristics were heat conduction coefficient, specific heat capacity and dry sand density, and for modeling the pile—reinforced concrete thermal characteristics given in Table 1.

The time step equal to 1 hour was chosen due to the Courant condition:

$$\Delta t \leq \frac{(\Delta x)^2}{4\left(\kappa/\rho c\right)_{max}}.$$

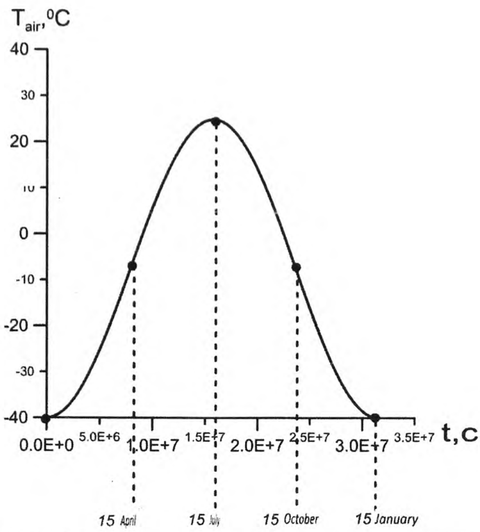


*Fig. 2.* The dependence of the ambient temperature on the time

*Table 1*

**Thermal and physical characteristics of the problem**

| Characteristics | κn, Wt/(m.°C) | ρn, kg/m3 | cn, J/(kg.°C) |
|---|---|---|---|
| Ground (n = g) | 0.512 | 1500 | 1020 |
| Pile (n = p) | 1.55 | 2200 | 840 |

**The analysis of the results**

This problem is typical, having no initial conditions [3], because the repetitive temperature course at the boundary makes the initial conditions have smaller or no impact on the temperature distribution at the moment of observation. If the surface temperature undergoes periodical changes for a long time, the soil has identical temperature changes. It takes 5 years to achieve a 'stationary periodic' state of the system in this problem. But even though the temperature field grows periodic for the fifth cycle, the temperature waves have the biggest impact on the area closer to the surface due to their little penetration depth (see Figs. 4, 5). Such a little penetration depth results from the temperature oscillation swings decreasing with the depth exponentially. It can be seen in Fig. 3 comparing temperature and time dependences at different depths.

Although the temperature at each point undergoes a periodic time change, thermal inertia contributes to phase difference between temperature oscillations at different depths. This difference between the temperature changes at the surface and under the pile makes a quarter of a period, i.e. 3 months. This difference between the temperature changes at the surface and at the 5 m depth makes 6 months. So, when it is summer at the surface, it is still spring at the 2 m depth, and it is winter at the 5 m depth. This is shown in Fig. 3, e.g. with $t = 1.4E + 8$. Such 'lagging behind' of the temperature perturbation is explained by the properties of the hyperbolic equation solution.

One of the most interesting results is that the temperatures range decreases with time at the pile points closer to the surface less than the temperatures range of the ground points far from the pile. It can be seen if we compare the temperature and time dependences at points (0.10) and (10.10) in Fig. 3. The difference between these ranges is ~0.9°C. It occurs because the volumetric heat capacity of the pile is bigger than that of the ground, so more heat is necessary to equal the temperature of the pile at the surface and the temperature of the ground. If the ambient temperature falls, vice versa, the pile temperature at the surface is higher than the ground temperature because the heat flow should be more intensive to cool the pile and again equal the temperatures. The deeper it goes, the more equal are the ranges until at some y value the temperature changes of the pile points grow bigger than those of the points far from the pile. This depth is influenced by the internal heat conduction defined by internal heat conduction coefficient κ, the influence of the external heat conduction [2], defined by the heat flow coefficient, decreases. Heat conduction $\kappa p > \kappa g$, so the deeper the pile goes, the cooler it becomes in a cool season, the warmer it becomes in a warm season. This process allows observing isotherm flex points close to the pile in Figs. 4 and 5.

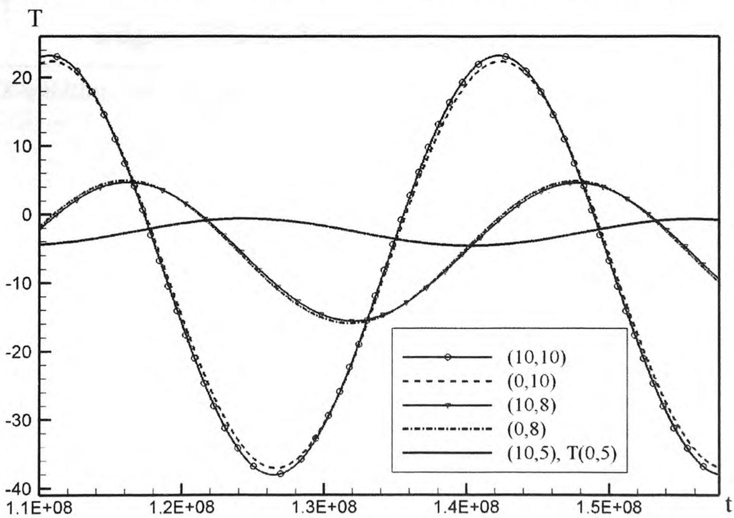


*Fig. 3.* The dependence of temperature on time at different points

Due to the bigger volumetric heat capacity and heat conduction coefficient the heat exchange is much more intensive between the environment and the pile than between the environment and the ground. With $\rho pcp/\rho gcg = 1.2$ and $\kappa p/\kappa g = 1.5$ both the heat flow swing and the heat exchange intensity in the pile are bigger than in the ground, 1.4 times in spring and summer, 1.6 times in autumn and winter. The heat flow density swing at the surface decreases with the distance from the pile (Fig. 6).

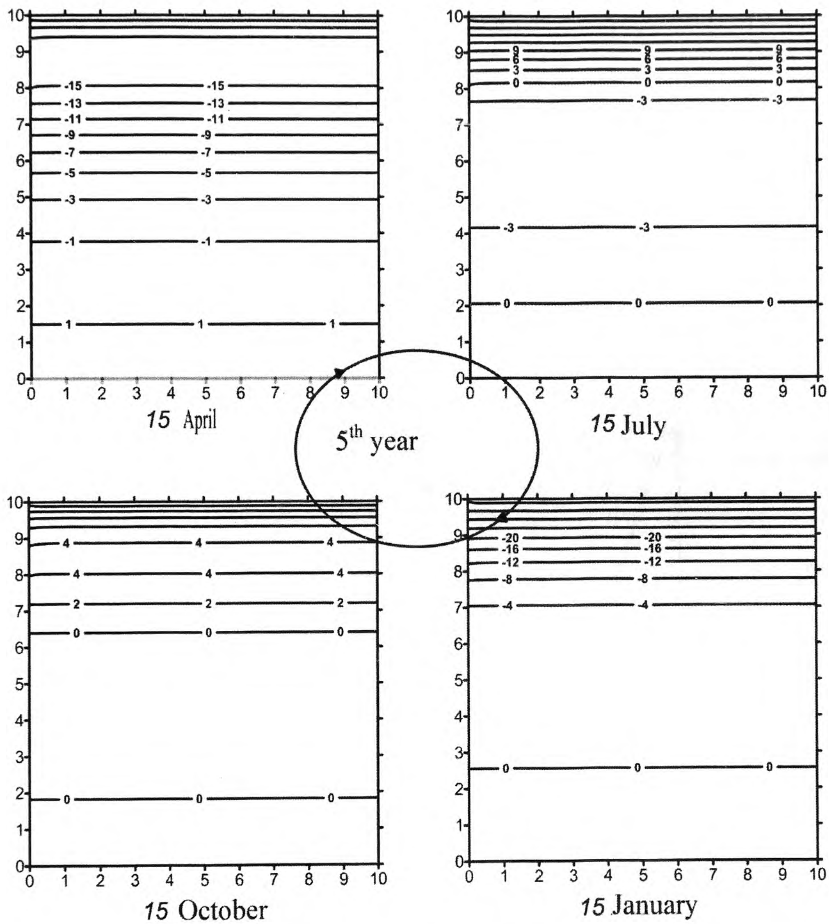


*Fig. 4.* Isotherms in the 5$^{th}$ year

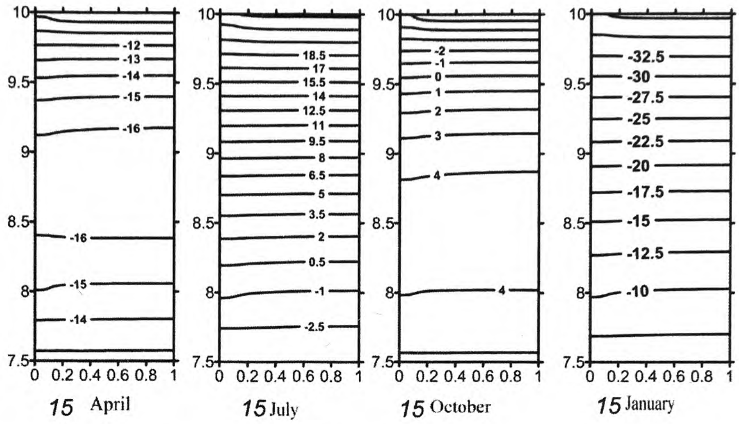


*Fig. 5*. Isotherms in the 5th year near the pile

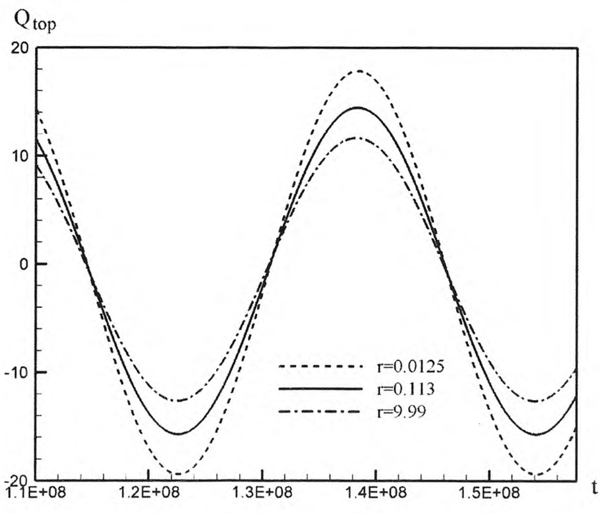


*Fig. 6*. The dependence of heat flow density and on time with y =10 M and r = 0.0125, 0.113, 9.99

**Conclusion.** The presented numerical experiment has shown that temperature waves and thermal construction influence the heat distribution in the ground. The conclusions are as follows:

1. It takes 5 years to achieve a "stationary periodic" state of the system in this problem.

2. Due to their small penetration depth temperature waves have the biggest impact on the temperature distribution in a small area at the surface of the ground.

3. Thermal inertia contributes to phase difference between temperature oscillations at different depths. The difference between the temperature changes at the surface and under the pile makes 3 months.

4. The pile heat exchange proves to be more intensive with the environment than with the ground.

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