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UDC 532.685

*MODELLING OF OIL DROPLET DYNAMICS IN A CONSTRICTED CAPILLARY TUBE**

ABSTRACT. External vibration (seismic) stimulation is one of the methods of enhanced oil recovery. Despite the fact that there are a lot of papers of both theoretical and experimental nature on the methods of vibration stimulation in fluid-saturated porous media, a number of issues remains open. For example, the mobilizing mechanism of oil droplets dispersed in the capillaries of the porous media is still poorly understood. This mechanism can be studied through mathematical modelling. The mathematical model can be based on hydrodynamics equations. Along with this, one can consider a simplified approach which uses the equation of droplet motion under the influence of the pressure gradient force and vibration force. This paper contains a numerical study of the dynamics of the droplet trapped in a constricted capillary tube under the vibration influence. The eigenfrequencies of the trapped droplets have been calculated at varying external static pressure gradient. We show that the mobilizing effect is dependent on the frequency of vibration. The analysis shows that this effect also depends on the droplet viscosity and of the vibration wave shape.

KEY WORDS. Oil droplet, vibration (seismic) stimulation, eigenfrequency.

Introduction

Methods of seismic stimulation for fluid-saturated porous media have recently attracted a lot of interest because they are among most economically feasible methods of enhanced oil production [1–6]. External vibration stimulation of porous medium is one of these methods, but the mechanism of pore-trapped oildrop mobilization is under researched.

This mechanism can be studied through mathematical modelling. The mathematical model can be based on the hydrodynamics equations, which are implemented in ANSYS FLUENT software. It is noteworthy that this approach requires significant computer capacities. Along with this, one can consider a simplified approach in which the equation of motion of the droplet is issued under a constant pressure gradient and vibration force. This approach was taken by the researchers in [7, 8].

*This research was supported by RF President Grant Council for Government Support to RF Leading Scientific Schools (Scientific School Grant-834.2012.1).

This research sets out to develop a methodology based on the model in [8] to find eigenfrequencies of trapped oil droplets. It goes on to study the influence of the wave shape and oil viscosity on seismic stimulation of trapped oil droplets.

Setting the task

Let there be a trapped oil droplet immersed in water in a constricted capillary tube at a constant pressure gradient. It is required to determine the parameters of vibration which will make the droplet pass the constriction.

Let the capillary be a cylinder with hydrophilic surface. The capillary constriction is sine-shaped, the dependence of the capillary radius on the longitudinal coordinate is described by the formula [7]:

$$r(z) = r_{\max}\left[1+\frac{1}{2}\left(\frac{r_{\min}}{r_{\max}}-1\right)\left(1+\cos\pi\frac{z}{L}\right)\right] \qquad (1)$$

where r_{min}, r_{max} are the minimum and the maximum radii of the tube, L is half-time (Fig.1).

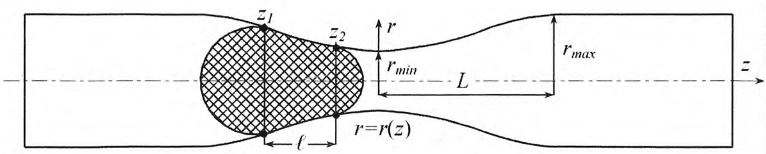

Fig. 1. Geometry of the constricted tube

The droplet is that of a nonwetting fluid, the drop-surface contact angle has zero curvature.

Model equation [8] becomes

$$\frac{d^2z_1}{dt^2}-\frac{2\sigma}{\rho_{oil}\ell}\left[\frac{1}{r_i(z_1)\sqrt{1+r_i'^2(z_1)}}-\frac{1}{r_i(z_1+\ell)\sqrt{1+r_i'^2(z_1+\ell)}}\right]+a(t)+\frac{\Delta P_{stat}}{\rho_{oil}\ell}= \\ -\frac{4\mu_{oil}}{\rho_{oil}\bar{r}_c^2\left[1+\frac{\mu_{oil}}{\mu_w}\frac{dh}{\bar{r}_c}\left(2+\frac{dh}{\bar{r}_c}\right)\right]}\frac{dz_1}{dt} \qquad (2)$$

where σ is surface tension, ρ_{oil}—oil gravity, μ_{oil}—oil viscosity, μ_w—water viscosity, ℓ—drop-surface contact zone length, ΔP_{stat}—static pressure gradient at length ℓ, dh—thickness of the wetting layer, $\bar{r}_c \equiv (r_{max}+r_{min})/2$.

The vibration force is calculated as

$$a(t)=A_0\cdot\sin(2\pi\nu t) \qquad (3)$$

where ν—frequency, t—time, A_0—amplitude of stimulation.

The vibration model can either be based on the longitudinal oscillations or on the cyclic change of pressure gradient. Then A_0 in (3) has the formula

$$A_0 = \begin{cases} a_0 & \text{- amplitude of capillary acceleration or;} \\ \Delta P_{osc} / \rho_{oil} \ell & \text{- specific pressure gradient} \end{cases} \tag{3}$$

Calculation results

Calculations for eigenfrequency of a trapped drop. Equation (2) helps to find eigenfrequencies ω_{eigen} of a drop trapped in a pore. This requires numeric solution for equation (2) without the addends representing the external force of vibration and viscosity. Viscosity has no influence on eigenfrequency; it determines damping of oscillation. Below we describe the approach to determine eigenfrequency.

We assume that a drop is trapped in the capillary. Let us calculate the coordinates of the three-phase contact lines z_1 и z_2 (Fig. 1). Coordinate z_1 can be found from the condition of equality of the static pressure gradient ΔP_{stat} and the balancing gradient of capillary pressure ΔP_{cap} which is calculated by the Laplace formula

$$\Delta P_{stat} = 2\sigma \left(\frac{1}{r_i(z_1)\sqrt{1 + r_i'^2(z_1)}} - \frac{1}{r_i(z_1 + \ell)\sqrt{1 + r_i'^2(z_1 + \ell)}} \right) \tag{4}$$

Coordinate z_2 is found as $z_2 = z_1 + \ell$ (Fig.1).

If the drop is slightly moved to the left, it starts producing longitudinal oscillations. The frequency of these oscillations can be taken as ω_{eigen}.

Thus, we are to solve the equation (2) at the given value of ΔP_{stat}, but the initial position of the drop is to be moved to the left (Fig. 1) from coordinates z_1 and z_2. The result of the equation describes the oscillations of the droplet with the frequency of ω_{eigen}.

We have studied the dependence of eigenfrequency ω_{eigen} from ΔP_{stat}.gradient. For example, Fig. 2 shows a curve calculated for the values from the experiment in [9]: $L = 12.1$ *mm*, $r_{max} = 4.23$ *mm*, $r_{min} = 0.12$ *mm*, $\ell = 11.8$ *mm*, $\rho_{oil} = 9\ 98$ *kg/m*^{3}, $\sigma = 0.016$ *N/m*, $dh = 0.001 \cdot r_{min}$, $\mu_{oil} = 0.44 \cdot 10^{-3}$ *Pa·s* и $\mu_w = 0.001$ *Pa·s*. Static pressure gradient was varying from 0 to $\Delta P_{crit} = 259.1$ Pa. Equation (2) is used to find ΔP_{stat}. gradient, at which the drop passes the constriction.

The above shows that the dependence of is ω_{eigen} from ΔP_{stat} gradient nonmonotone.

It can be expected that the vibration force mobilizing the trapped drops is dependent on its frequency, the minimum of this force being registered at the vibration frequency close to that of the drop eigenfrequency. To demonstrate the results of the numeric analysis we have shown the dependence of the vibration stimulation force on the frequency at the static pressure gradient values of $\Delta P_{stat} = 95$ Pa and $\Delta P_{stat} = 171$ Pa.

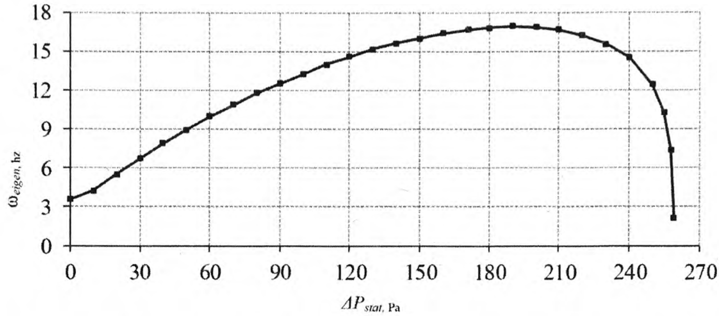

Fig. 2. The dependence of the drop eigenfrequency on the static pressure gradient

As it can be seen from Fig. 3, the vibration force minimum is registered at the frequency of 11 Hz at ΔP_{stat} = 95 Pa and 15 Hz at ΔP_{stat} = 171 Pa. The eigenfrequency of the drop (Fig. 2) is 13 Hz and 16.6 Hz at ΔP_{stat} = 95 Pa and ΔP_{stat} = 171 Pa respectively. So, the optimum frequency of external vibration stimulation is close to the frequency of free oscillations at the given external static pressure gradient.

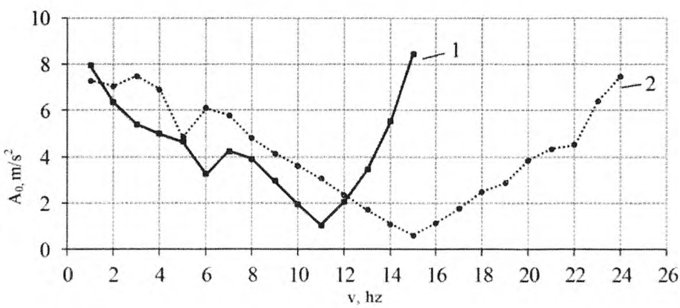

Fig. 3. The dependence of the vibration stimulation force on the frequency (ΔP_{stat} = 95 *Pa*—curve 1, ΔP_{stat} = 171 *Pa*—curve 2)

The influence of the oil viscosity on the drop mobilization. Water-oil displacement from the layer is effective, when the oil viscosity is not more than 7–10 times higher than the viscosity of water [10]. It is important to study the influence of oil viscosity on vibration stimulation within the said viscosity range. Calculations are based on the data for the experiment in [9] at ΔP_{stat} = 171 Pa and the frequencies of ν = 10, 15, 20 Hz. The frequency of ν = 15 Hz corresponds to the minimum mobilizing force (curve 2 Fig. 3), frequencies ν = 10 and 20 Hz are in its vicinity. The results obtained are set out in Fig. 4.

Fig. 4 shows that the vibration amplitude steadily rises at the increase of the viscosity, but throughout the range of viscosity values the minimum amplitude is registered at 15 Hz.

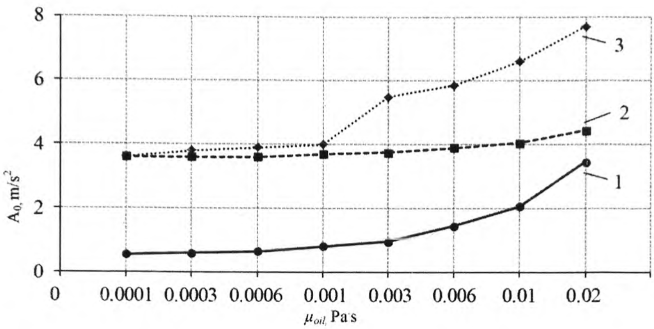

Fig. 4. The dependence of the vibration stimulation force on the oil viscosity for different frequencies (ν =15 Hz—curve 1, ν =10 Hz—curve 2, ν =20 Hz—curve 3) at the static pressure gradient ΔP_{stat} = 171 Pa.

The influence of the wave shape on the drop mobilization. It is important to study the influence of the shape of the wave on the oil drop mobilization. We will consider four dependencies $a(t)$ in equation (2) which are shown in Fig 5.

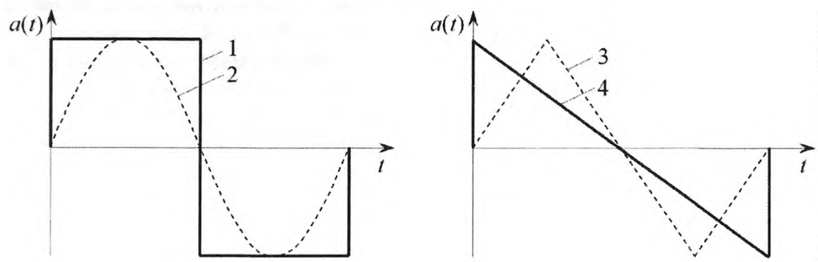

Fig. 5. Wave shapes

By solving equation (2) we calculate the dependences of the vibration stimulation force on the vibration frequency for the values of curve 2 at Fig. 3 and the dependences $a(t)$ at Fig. 5. The results obtained are set out in Fig. 6.

Figure 6 shows that for the dependences 1–4 in Fig. 5, which were used in the calculations, the stimulation force is more dependent on the frequency than on the wave shape. We can add that the change of the stimulation phase, i.e. substitution of '+' for '-' in equation (2), does not change the curves 1–4 in Fig. 6.

Conclusion

The eigenfrequency of a trapped droplet shows nonmonotone dependence on the static pressure gradient.

The mobilization amplitude is minimum at the frequency of vibration close to the eigenfrequency of the droplet. It gradually increases with the growth of oil viscosity values, but is almost independent of the vibration wave shape.

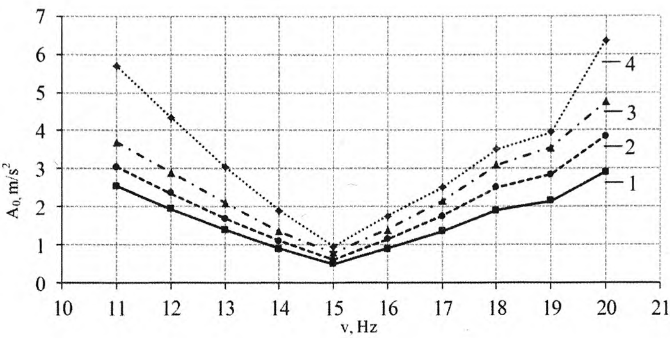

Fig. 6. The dependence of the vibration mobilization force on the frequency at different wave shapes (Fig. 5)

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