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*INVESTIGATION OF BUBBLE CLUSTER DYNAMICS**

ABSTRACT. The behaviour of a single bubble in compression waves in a group of bubbles may differ from the behaviour of a single bubble in infinite liquid due to the collective hydrodynamic interaction. The paper presents a mathematical model describing the dynamics of a system of bubbles with changing radii in infinite liquid with regard to its compressibility and viscosity. The model also takes into account the hydrodynamic interaction between bubbles. Heat exchange between gas bubbles and liquid is considered by a two-temperature model. It is given calculation for heat flux per a bubble surface unit, allowing to describe heat exchange between gas bubbles and liquid in a sufficiently wide range of the fluid's pressure and temperature values. It is used uses a numerical simulation of nonlinear dynamics of bubble clusters with different configuration under the influence of a compression wave. Behaviour of a single bubble in a group of bubbles under the action of compression waves is investigated. It is shown that in certain conditions some bubbles are subject to significant degrees of compression, and as a consequence, to high pressures and temperatures. The example of a cluster with three nested dodecahedrons shows that a cluster configuration can have a considerable impact on its dynamics.

KEY WORDS. Bubble cluster, compression waves, a dynamic model of a bubble system, bubble-liquid heat exchange.

Introduction

The interest in the study of bubble cluster dynamics is caused by the fact that the behaviour of a single bubble in compression waves in a group of bubbles may differ from the behaviour of a single bubble in infinite liquid due to the collective hydrodynamic interaction. There are various approaches and models which take into account the above mentioned interaction of bubbles in a cluster [1–5]. The smaller the distance between bubbles compared to their size, the more accurately hydrodynamic bubble interaction should be estimated. For example, models [2–4] have the second and third orders of accuracy respectively, and a model with an arbitrary order of accuracy does not exist yet.

The studies [6, 7] present empirical data where the production of neutrons and tritium nuclei is registered when bubble clusters are exposed to acoustic influence.

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However, it has been noticed that the emergence of neutron emission depends on the shape of a cluster.

The purpose of the study is a computer-generated simulation of bubble cluster dynamics depending on the cluster configuration.

Problem statement

Consider a bubble cluster consisting of N-gas bubbles as a compact structure in a sufficiently large amount of liquid [1–4, 8, 9]. It is evident that in a close-packed cluster hydrodynamic interaction may lead to a strong deformation of bubble surface which is difficult to estimate theoretically. Therefore we are going to consider a sparse cluster in which bubbles could keep their spherical shape.

Simulating bubble cluster dynamics, we use the following assumptions:

The liquid is incompressible $\rho_l = const$.

The liquid flow is potential.

The cluster is sparse.

The temperature of liquid is constant due to a high heat capacity of liquid compared to gas $T_l = const$.

The gas in bubbles is perfect with an adiabatic exponent γ.

Gas pressure in bubbles is homogeneous.

The system of equations for cluster dynamics will include equations of bubbles' motion in liquid with regard to their hydrodynamic interaction [4]. We will also take into account bubble-liquid heat exchange and keep the notations [4].

The flow potential of incompressible ideal liquid φ satisfies Laplace's equation:

$$\Delta\varphi = 0, \tag{1}$$

with j-bubble surface boundary conditions and far from the cluster:

$$\vec{n}_j \cdot \vec{\nabla}\varphi = \dot{R}_j + \dot{\vec{p}}_j \cdot \vec{n}_j \text{ with } \vec{r} = \vec{p}_j + R_j\vec{n}_j, \tag{2}$$

$$\left|\vec{\nabla}\varphi\right| \to 0 \text{ with } \left|\vec{r} - \vec{p}_j\right| \to \infty, \tag{3}$$

The perfect gas state equation is as follows:

$$P_{gn} = \frac{\rho_{gn} T_{gn} c_{pg} (\gamma - 1)}{\gamma}. \tag{4}$$

The equations of bubble motion are as follows:

$$\begin{gathered}
\left(1 - \frac{\dot{R}_n}{c_l}\right) R_n \ddot{R}_n + \left(\frac{3}{2} - \frac{\dot{R}_n}{2c_l}\right)\dot{R}_n^2 - \frac{P_n}{\rho_l}\left(1 + \frac{\dot{R}_n}{c_l}\right) - \frac{R_n}{\rho c_l}\frac{dP_n}{dt} \\
= \frac{\dot{\vec{p}}_n^{\,2}}{4} - \sum_{l=1, l \neq n} \left\{ \frac{R_l^2 \ddot{R}_l + 2R_l \dot{R}_l^2}{d_{nl}} + \frac{R_l^2}{2d_{nl}^3}(\vec{p}_n - \vec{p}_l)\cdot\left(R_l \ddot{\vec{p}}_l + \dot{R}_l \dot{\vec{p}}_n + 5\dot{R}_l \dot{\vec{p}}_l\right) - \right. \\
\left. - \frac{R_l^3}{4d_{nl}^3}\left[\dot{\vec{p}}_l \cdot \left(\dot{\vec{p}}_n + 2\dot{\vec{p}}_l\right) + \frac{3}{d_{nl}^2}\left[\dot{\vec{p}}_l \cdot (\vec{p}_l - \vec{p}_n)\right]\left[(\vec{p}_n - \vec{p}_l)\cdot\left(\dot{\vec{p}}_n + 2\dot{\vec{p}}_l\right)\right]\right]\right\},
\end{gathered} \tag{5}$$

$$\frac{1}{3}R_n\ddot{\vec{p}}_n + \dot{R}_n\dot{\vec{p}}_n = \frac{\vec{F}_n}{2\pi\rho_l R_n^2} + \sum_{l=1,l\neq n}\left\{\frac{\vec{p}_n - \vec{p}_l}{d_{nl}^3}\left(R_n R_l^2 \ddot{R}_l + 2R_n R_l \dot{R}_l^2 + \dot{R}_n \dot{R}_l R_l^2\right) - \right.$$
$$-\frac{R_l^2}{2d_{nl}^3}\left[R_n R_l \ddot{\vec{p}}_l + \left(\dot{R}_n R_l + 5R_n \dot{R}_l\right)\dot{\vec{p}}_l\right] + \tag{6}$$
$$\left. + \frac{3R_l^2(\vec{p}_n - \vec{p}_l)}{2d_{nl}^5}\left[(\vec{p}_n - \vec{p}_l)\cdot\left[R_n R_l \ddot{\vec{p}}_l + \left(\dot{R}_n R_l + 5R_n \dot{R}_l\right)\right]\right]\right\},$$

where equation (5) describes radial bubble pulsations of n-bubble and equation (6) describes its translational motion.

P_n denotes the following expression [4, 10]:

$$P_n = P_{gn} - \frac{2\sigma}{R_n} - \frac{4\eta\dot{R}_n}{R_n} - P_0 - P_{ex}(\vec{p}_n), \tag{7}$$

where P_{gn} is gas pressure in a bubble, σ—surface tension coefficient, η—coefficient of dynamic viscosity, P_0—hydrostatic pressure, P_{ex}—external acoustic pressure within the range of n-bubble.

Gas pressure in a bubble can be expressed by means of an equation of state (4) with the following initial parameters:

$$P_{gn} = \left(P_0 + \frac{2\sigma}{R_{n0}}\right)\left(\frac{R_{n0}}{R_n}\right)^{3\gamma} \tag{8}$$

where R_{n0} is n-bubble radius in equilibrium.

The interphase force is viscous friction force proposed by Levich [11]:

$$\vec{F}_n = \vec{F}_{dn} = -12\pi\eta R_n\left(\dot{\vec{p}}_n - \sum_{l=1,l\neq n}^{N}\vec{v}_{ln}\right), \tag{9}$$

where v_{ln} is liquid velocity generated by l-bubble in the centre of n-bubble. With an accuracy up to terms $1/(d_{jl})^3$, the velocity v_{ln} is expressed as follows:

$$\vec{v}_{ln} = \frac{R_l^2\dot{R}_l(\vec{p}_n - \vec{p}_l)}{d_{nl}^3} + \frac{R_l^3}{2d_{nl}^3}\left\{\frac{3(\vec{p}_n - \vec{p}_l)}{d_{nl}^2}\left[\dot{\vec{p}}_l\cdot(\vec{p}_n - \vec{p}_l)\right] - \dot{\vec{p}}_l\right\}. \tag{10}$$

Describing gas bubble-liquid heat exchange, we are going to use a two-temperature model in terms of which the equation of gas pressure inside bubbles is as follows:

$$\frac{dP_{gn}}{dt} = \frac{3(\gamma - 1)}{4\pi R_n^3}q_n - \frac{3\gamma P_{gn}}{R_n}\dot{R}_n. \tag{11}$$

The expression for heat flux per n-bubble surface unit q_n is the following [12]:

$$q_n = 2\pi R_n \lambda_g \,\mathrm{Nu}_{gn}\left(T_l - T_{gn}\right), \tag{12}$$

$$\mathrm{Nu}_{gn} = \begin{cases} \sqrt{\mathrm{Pe}_{gn}}, & \mathrm{Pe}_{gn} \geq 100 \\ 10, & \mathrm{Pe}_{gn} < 100 \end{cases},$$

$$\mathrm{Pe}_{gn} = 12(\gamma - 1)\frac{T_l}{\left|T_{gn} - T_l\right|}\frac{R_n\,|\dot{R}_n|}{D_{gn}}, \tag{13}$$

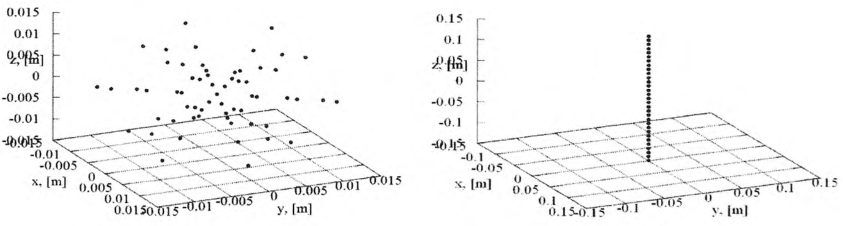
$$D_{gn} = \frac{\lambda_g}{c_{pg}\rho_{gn}},$$

where Nu_{gn} is the Nusselt number, Pe_{gn}—the Péclet number, T_{gn}—gas temperature in n-bubble, D_{gn}—thermal conductivity coefficient of gas, λ_g—gas heat conductivity coefficient, c_{pg}—gas heat capacity at constant pressure, ρ_g—gas density.

To solve the system of equations (5)–(10), it is necessary to set initial coordinates, velocities, radii, radial bubble velocities, gas pressure in bubbles and the law of liquid pressure variation at a distance from a bubble cluster.

Calculation data

The dynamics of bubble clusters with different configuration has been investigated. The results of the study are illustrated by two configurations (Fig. 1).



a b

Fig. 1. A cluster with three nested dodecahedrons (a) and a linear cluster (b).

In the first case the cluster consisted of three concentrically nested dodecahedrons (Fig. 1 (a)), in the second case the cluster was a linear streamer (Fig. 1 (b)). Suppose the dodecahedrons are nested in spheres of *5, 10, 15 mm* radii respectively. The initial length of the linear cluster is *30 mm*. The initial radii of the central bubble and peripheral bubbles are the same, they equal $R_0 = 0.25$ *mm*. The total number of bubbles in both cases is the same—*61*.

A step-type compression wave was considered external influence P_{ex} in (7) when fluid pressure away from the cluster increased in the following way:

$$P_{ex}\left(\vec{p}_n\right) = \begin{cases} P_0, & t < 0, \\ P_0 + \Delta P, & t \geq 0, \end{cases} \tag{14}$$

where the pressure difference ΔP was equal to *0.3 MPa*.

The initial pressure is atmospheric pressure: $P_0 = 0.1$ *MPa*, the temperature of liquid is $T_l = 293$ *K*.

Parameters of liquid and gas were the following: liquid—*60%* glycerol solution in water (the speed of sound is $c_l = 1500$ *m/s*, density is $\rho_l = 1156\ kg/m^3$, kinematic viscosity is $\nu = 9.52 \cdot 10^{-6}\ m^2/s$, surface tension coefficient is $\sigma = 7 \cdot 10^{-2}\ N/m$), gas—air (adiabatic exponent is $\gamma = 1.4$, specific heat capacity at constant pressure is $c_{pg} = 1007\ J/(kg \cdot K)$, heat conductivity coefficient is $\lambda_g = 0.0258\ W/(m \cdot K)$).

To nondimensionalize the parameters, take the value $t^* = R_0 \cdot (\rho_l/P_0)^{1/2} = 2.7 \cdot 10^{-5} s$ as characteristic time which corresponds to the time of collapse of an empty sphere with radius R_0 in liquid with density ρ_l and pressure P_0.

Figure 2 shows the calculation data for a cluster consisting of three dodecahedrons: dependencies of non-dimensional radius R/R_0 and non-dimensional pressure P/P_0 on non-dimensional time t/t^* for the central (a) and peripheral (b) bubbles. It can be seen that the central bubble is exposed to significant amplitudes of compression and, as a consequence, a significant pressure increase as compared to the peripheral bubbles. The maximum temperature for the central bubble was ≈2000 K. The characteristic peak length is $\sim 10^{-8}$–10^{-9} s.

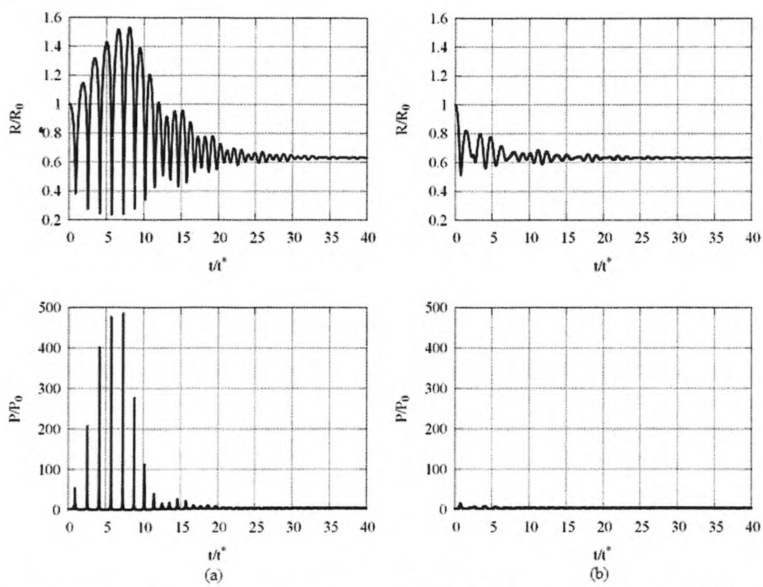


Fig. 2. The dependence on time of pressure and radius of the central (a) and peripheral (b) bubbles in case of a bubble cluster of three nested dodecahedrons.

Figure 3 shows the calculation data for a linear cluster: dependencies of non-dimensional radius R/R_0 and non-dimensional pressure P/P_0 on non-dimensional time t/t^* for the central (a) and peripheral (b) bubbles. In this case the central bubble is compressed less than in the case of three dodecahedrons. The maximum pressure both for the central and peripheral bubbles does not exceed 22 atm. The amplitude of oscillations during modelling time does not exceed the initial radius of the bubble.

In both cases bubble oscillations die out due to interphase heat exchange (11)–(13) and sound radiation.

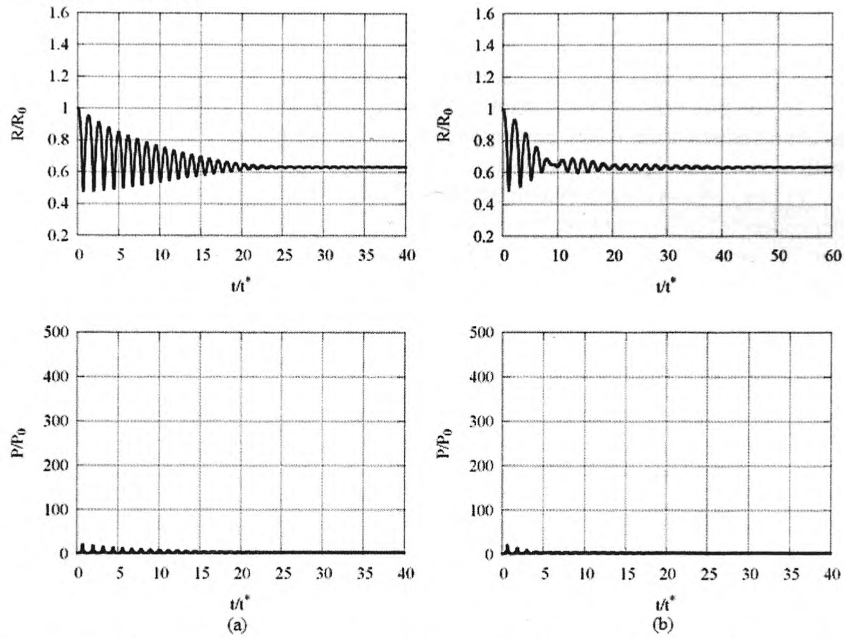


Fig. 3. The dependence on time of pressure and radius of the central (a) and peripheral (b) bubbles in case of a streamer.

Conclusions

The dynamics of bubble clusters with different configuration has been investigated in the paper.

A numerical simulation of nonlinear dynamics of bubble clusters with different configuration under the influence of a step-type compression wave has been performed. It has been shown that in a cluster, whose bubbles are situated at vertexes of a dodecahedron and in its centre, the central bubble is exposed to significant amplitudes of compression and, as a consequence, a significant pressure and temperature increase as compared to the peripheral bubbles.

A cluster configuration can have a considerable impact on its dynamics. Pressures in a cluster with dodecahedrons exceed the pressures in a streamer manifold, which is in qualitative agreement with experimental observations.

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