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DRYING SIMULATION OF A THIN LAYER OF HUMID POROUS MEDIUM

ABSTRACT. The process of heating a porous medium which contains liquid and is limited by walls permeable to mass and heat flows on one side and impermeable on the other is considered. The system of equations describing the heat and mass transfer during drying of a thin layer of a porous medium is introduced. The numerical solutions showed that the drying time for a thin layer is usually less than the time of formation of the dry zone of the same size in an infinite medium. However, this dependence can be violated in two cases: if the relative mass concentration of vapor at the outer boundary is close to 1, a high temperature is set in due course in the wet zone reducing the heat input from the outside; if the initial temperature of a medium is close to the steam point, the heat input from the depths is promptly cut off.

KEY WORDS. Porous medium, moisture, drying, belt.

The drying of belt porous materials such as paper, cloth, and rubber are widespread in many processes. In this connection, it is necessary to take into account the boundary effect on the dynamics of temperature and concentration fields. In [1], the processes of high-temperature drying of capillary-porous materials are considered. They are accompanied by the deepening of the evaporation zone during drying. Objective assessment of the description adequacy of the external transfer using certain criterion dependency leads to the permissible error value—20% [2]. Taking into account the research [3], it seems justified not to make more stringent requirements for the rigor of mathematical formulation of the internal transfer, but deliberately simplify the problem. In [4], it is shown that the activation of hydrodynamic regimes is one of the ways to improve the drying process. At the same time it should be noted that the self-similar solutions obtained in [5] can be used to estimate the approximate and computational solutions.

Dynamics of heat and mass transfer. Let us consider the case of heating of the porous medium containing liquid and localized on one side by a permeable boundary and on the other side—by an impervious (to heat and mass flow) boundary (Fig. 1). We assume valid assumptions: single-temperature process, i.e. the temperature of the porous medium and the saturating fluid at each point match, the solid matrix and the fluid are incompressible and stationary, porosity is constant, drying occurs due to vapor diffusion and heat transfer—due to thermal conduction through the liquid and the solid matrix, the pressure of vapor-gas mixture is homogeneous and equal to atmospheric pressure. In proportion to the drying of the material from the outer boundary, a dry zone is formed, free of water in the liquid state.

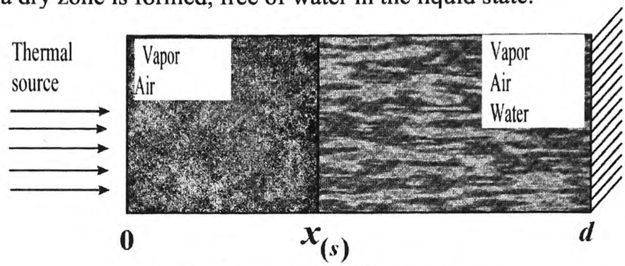

Fig. 1. The geometry of the problem.

The self-similar solutions set up in [1], can be applied either when the porous medium has an infinite length or time is $t < d^2/4\kappa$, where d—typical dimensions of the porous medium, κ—thermal diffusivity, i.e. until heat propagation process starts affecting the far boundary of the medium. In practice, all the materials subjected to drying have a finite length. In this connection, it is necessary to take into account the limited size of bodies and their influence on the dynamics of the boundaries of the temperature and concentration fields. At the same time, it should be noted that the above self-similar solutions can be used to estimate the approximate and computational solutions.

The process of heat and mass transfer will be described in the framework of continuum mechanics [6] with the system of equations taken from [5].

$$\frac{\partial \rho_v}{\partial t} = D\frac{\partial^2 \rho_v}{\partial x^2},$$

$$\rho c\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial x^2}, \quad \left(0 < x < x_{(s)}\right) \tag{1}$$

$$\frac{\partial S_1}{\partial t} = \frac{(1 - S_l)}{\rho_1}D\frac{\partial^2 \rho_v}{\partial x^2},$$

$$\rho c\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial x^2} + mL\rho_1\frac{\partial S_1}{\partial t},$$

$$\rho_v = \frac{\mu_v p_*}{RT}exp\left(-\frac{T_*}{T}\right). \qquad \left(x_{(s)} < x < \infty\right) \tag{2}$$

The terms of mass and heat balance at the interface between humid and dry zones:

$$D\left(\frac{\partial \rho_v}{\partial x}\right)^-_{(s)} - \left(1 - S_{1(s)}\right)D\frac{\rho_v}{T}\left(\frac{T_*}{T} - 1\right)\left(\frac{\partial T}{\partial x}\right)^+_{(s)} = \rho_1 S_{1(s)}\dot{x}_{(s)},$$

$$\left(\lambda\frac{\partial T}{\partial x}\right)^+_{(s)} - \left(\lambda\frac{\partial T}{\partial x}\right)^-_{(s)} = mL\rho_1 S_{1(s)}\dot{x}_{(s)}. \tag{3}$$

Initial and boundary conditions can be written as

$$T = T_0,\ S_l = S_{l0}\ (t = 0)$$
$$T = T_e,\ \rho_v = \rho_{ve}\ (x = 0) \tag{4}$$
$$\frac{dT}{dx} = 0\ (x = d)$$

The formulated boundary value problem belongs to a class of nonlinear boundary value problems of mathematical physics. Because this problem is defined in the areas with unknown moving boundary, we use the method of capturing the front in the grid node [7], since it can accurately track the movements of the desired border. The basic idea of this method lies in the fact that unless the temperature perturbation reaches the right border, the distribution of moisture content and temperature are approximated with a self-similar solution. After that, the next step of a time grid is calculated in the solution of the problem, together with the equilibrium values of the moisture content and temperature.

Numerical implementation. We proceed to the description of the difference scheme. On the interval $[0,d]$, where $d = x_{(s)}(t^*)$, t^*—the final time, we construct a regular spatial grid, consisting of nodes with step h, so that the point $x_{(s)}(0)$ coincides with one of the nodes:

$$x_j = jh,\ j = \overline{0,n},\ x_0 = 0,\ x_n = d.$$

An irregular grid is constructed on the interval $[0,t^*]$.

$$t_i = t_{i-1} + \tau_i,\ i = \overline{1,i_0},\ t_{i0} = t_*,\ \tau_i > 0.$$

A grid step of time τ_i is chosen in the way that for each time step the front of the phase transition moves along x coordinate in exactly one step, i.e. $x_{(s)}(t_i) - x_{(s)}(t_{i-1}) = h$.

We write the equations (1) and (2) in the form of finite-difference equations [8].

For $j = \overline{1,i-1}$

$$\frac{\rho_j - \hat{\rho}_j}{\tau_i} = D\frac{\rho_{j+1} - 2\rho_j + \rho_{j-1}}{h^2},$$
$$\frac{T_j - \hat{T}_j}{\tau_i} = \kappa\frac{T_{j+1} - 2T_j + T_{j-1}}{h^2}, \tag{5}$$

for $j = \overline{i+1,n-1}$

$$\rho_1\frac{S_j - \hat{S}_j}{\tau_i} = \frac{D}{h^2}\left[\left(1 - \frac{\hat{S}_{j+1} + \hat{S}_j}{2}\right)\left(\hat{\rho}_{j+1} - \hat{\rho}_j\right) - \left(1 - \frac{\hat{S}_j + \hat{S}_{j-1}}{2}\right)\left(\hat{\rho}_j - \hat{\rho}_{j-1}\right)\right],$$
$$\frac{T_j - \hat{T}_j}{\tau_i} = \kappa\frac{T_{j+1} - 2T_j + T_{j-1}}{h^2} + \frac{mL\rho_l}{\rho c}\frac{\Delta\hat{S}_j}{\tau_i}. \tag{6}$$

Hereinafter, the symbol '^' indicates the value of the previous sacrificial layer. We suppose, that $\Delta S = S_j - \hat{S}_j$ initial-boundary conditions in the discrete form will be as follows

$$T_j^0 = T_0,\ S_j^0 = S_0,\ \left(j=\overline{0,n}\right)$$
$$T_0^j = T_e,\ \rho_0^j = \rho_e,\ (j>0) \tag{7}$$
$$\frac{T_n - T_{n-1}}{h} = 0.$$

The system (3) will be as follows

$$\begin{gathered} D\left(\frac{\partial \rho_v}{\partial x}\right)_i^- - (1-S_i)D\left(\frac{d\rho_v}{dT}\right)_i^+\left(\frac{\partial T}{\partial x}\right)_i^+ = \rho_l S_i \dot{x}_i, \\ ((1-m)\lambda_m + m\lambda_l S_i)\left(\frac{\partial T}{\partial x}\right)_i^+ - (1-m)\lambda_m\left(\frac{\partial T}{\partial x}\right)_i^- = mL\rho_l S_i \dot{x}_i. \end{gathered} \tag{8}$$

Solving it relating to S_i and $(\partial T/\partial x)_i^+$, we will have

$$aS_i^2 + bS_i + c = 0, \qquad a = \frac{DL}{\lambda_l}\left(\frac{d\rho_v}{dT}\right)_i^+ - 1,$$
$$b = \frac{D}{\rho_l \dot{x}_i}\left(\frac{\partial \rho_v}{\partial x}\right)_i^- - \frac{(1-m)\lambda_m}{m\lambda_l} + \left(\frac{D}{\rho_l \dot{x}_i}\frac{(1-m)\lambda_m}{m\lambda_l}\left(\frac{\partial T}{\partial x}\right)_i^- - \frac{DL}{\lambda_l}\right)\left(\frac{d\rho_v}{dT}\right)_i^+,$$
$$c = \frac{D}{\rho_l \dot{x}_i}\frac{(1-m)\lambda_m}{m\lambda_l}\left(\left(\frac{\partial \rho_v}{\partial x}\right)_i^- - \left(\frac{d\rho_v}{dT}\right)_i^+\left(\frac{\partial T}{\partial x}\right)_i^-\right), \tag{9}$$
$$\left(\frac{\partial T}{\partial x}\right)_i^+ = \frac{mL\rho_l \dot{x}_i S_i + (1-m)\lambda_m\left(\frac{\partial T}{\partial x}\right)_i^-}{(1-m)\lambda_m + m\lambda_l S_i}.$$

The derivatives in (9) have the form in the finite-difference representation

$$\left(\frac{\partial \rho_v}{\partial x}\right)_i^- \approx \frac{\hat{\rho}_{j-1} - \hat{\rho}_{j-2}}{h},\ \left(\frac{\partial T}{\partial x}\right)_i^- \approx \frac{\hat{T}_{j-1} - \hat{T}_{j-2}}{h},\ \dot{x}_i \approx \frac{h}{\tau_i}. \tag{10}$$

At each time step, it is reasonable to implement the method of simple iteration to the constructed difference scheme (5)–(7), (9), (10), which is a system of nonlinear algebraic equations. The essence of the method is as follows.

The iteration counter is specified with the value $s = 0$, the values $T_j^s = T_j$, $S_j^s = S_j$, $\rho_j^s = \rho_j$ $(j=\overline{0,n})$ and time step $\tau_0 = (dh)/(2\kappa\xi_{(s)}\xi_{(\infty)})$ of the self-similar solution. Then we increment the iteration counter, solve the system (9) relating to S_i^s, and then find a new approximation T_j^{s+1} based on initial and boundary conditions (7). In this system the equations (5) and (6) can be written as:

$$T_{j+1}^{s+1}-\left(2+\frac{\rho_{\text{r}}c_{\text{r}}}{\lambda_{\text{r}}}\frac{h^2}{\tau_i}\right)T_j^{s+1}+T_{j-1}^{s+1}=-\frac{\rho_{\text{r}}c_{\text{r}}}{\lambda_{\text{r}}}\frac{h^2}{\tau_i}T_j^s \qquad (11)$$

$$\rho_{j+1}^{s+1}-\left(2+\frac{h^2}{D\tau_i}\right)\rho_j^{s+1}+\rho_{j-1}^{s+1}=-\frac{h^2}{D\tau_i}\rho_j^s \qquad (12)$$

$$T_{j+1}^{s+1}-\left(2+\frac{(1-m)\rho_{\text{r}}c_{\text{r}}+m\rho_1c_1S_j^s}{(1-m)\lambda_{\text{r}}+m\lambda_1S_j^s}\frac{h^2}{\tau_i}\right)T_j^{s+1}+T_{j-1}^{s+1}=$$
$$=-\frac{(1-m)\rho_{\text{r}}c_{\text{r}}+m\rho_1c_1S_j^s}{(1-m)\lambda_{\text{r}}+m\lambda_1S_j^s}\frac{h^2}{\tau_i}\left(T_j^s+\frac{mL\rho_l}{\rho c}\frac{\Delta S_j^s}{\tau_i}\right) \qquad (13)$$

$$S_j=\hat{S}_j+\frac{\tau_iD}{\rho_1h^2}\left[\left(1-\frac{\hat{S}_{j+1}+\hat{S}_j}{2}\right)\left(\hat{\rho}_{j+1}-\hat{\rho}_j\right)-\left(1-\frac{\hat{S}_j+\hat{S}_{j-1}}{2}\right)\left(\hat{\rho}_j-\hat{\rho}_{j-1}\right)\right] \qquad (14)$$

The equations (11)–(13) are solved implicitly, and (14)—explicitly. Once the temperature distribution (13) is found, the values ρ_j^{s+1} are determined by the following formula

$$\rho_j^{s+1}=\frac{\mu_{\text{v}}p_*}{RT_j^{s+1}}exp\left(-\frac{T_*}{T_j^{s+1}}\right) \qquad (15)$$

The next time step is defined as the geometric mean of the moving boundary conditions (8)

$$\tau_i^{s+1}=\sqrt{\tau_1\tau_2},\qquad \tau_1=\frac{h^2\rho_1S_{\text{i}}}{D\left[\left(\rho_{i-1}^s-\rho_{i-2}^s\right)-\left(1-S_i\right)\left(\rho_{i+1}^{s+1}-\rho_i^{s+1}\right)\right]},$$
$$\tau_2=\frac{mh^2\rho_1S_{\text{i}}}{\left((1-m)\lambda_{\text{r}}+m\lambda_1S_{\text{i}}\right)\left(T_{i+1}^{s+1}-T_i^{s+1}\right)-(1-m)\lambda_{\text{r}}\left(T_{i-1}^s-T_{i-2}^s\right)}. \qquad (16)$$

We repeat these steps to achieve the desired accuracy for pressure and temperature, and then proceed to the next time step.

Decision analysis. Figure 2 shows the profiles of temperature (a), volumetric moisture content (b), comparison of the position of a moving boundary coordinates $x_{(s)}$ (c) and the average moisture content of the material S_{INT} (d) under the following external parameters defining the initial state of the porous medium: $T_e = 160°C$, $k_{ve} = 0$, $T_0 = 20°C$, $S_{l0} = 0.5$, the half-layer d = 5mm. The remaining parameters are chosen by analogy with [5] from [9, 10]. The solution obtained for a self-similar problem is shown in a dashed line, the problems in the final layer—a solid line. It is seen that the temperature distribution in a humid area for the final layer becomes almost uniform over time and exceeds the temperature of a moving boundary in the case of self-similar problem due to the rapid heating of the restricted area. This, in its turn, leads to the equalization of vapor density in the depth of the material weakening and diffusion.

Therefore, the maximum moisture content of the humid zone at the border is less important compared to the self-similar solution. Figure 2c shows that the drying of the material in the final layer is much smaller (about 20%) than the formation of the dry zone of the same size in a semi-infinite, due to the lack of heat outflow deep into the material. Temperature equalization in the humid area occurs in a time of 200 sec, and warming the entire humid zone as a whole—for 400 sec. Further, all of the heat supplied to the moving boundary goes to the evaporation of water at a steady temperature of about 77°C. The vapor transfer in the humid area ends by the time. The change of the average moisture content over time is shown in Fig. 2d, which illustrates that it decreases more slowly. This indicates a decrease in the intensity of the drying process with time.

Integrated set of solutions is shown in Fig. 3 as a dependence of drying time on initial-boundary conditions. When an outside temperature T_e is changed (Fig. 3a) between 30 ÷ 160°C, the time of material drying is 1.2–1.3 times less than the formation of the dry zone of the same size in case of a self-similar formulation, which is associated with less input.

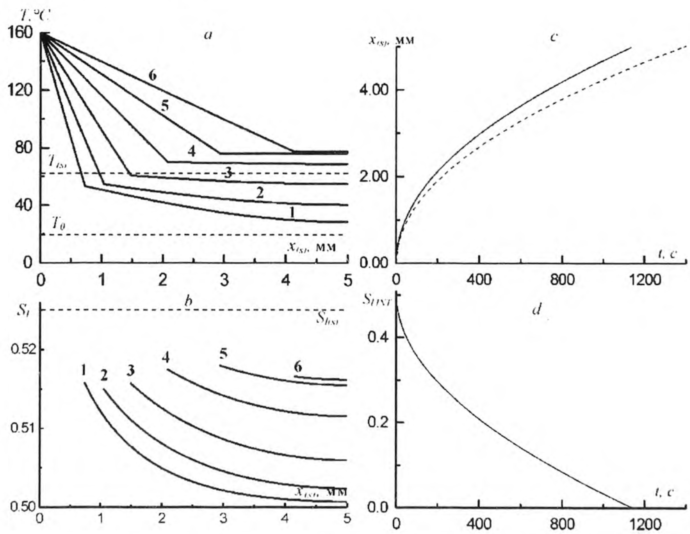

Fig. 2. Temperature profiles (a), volumetric moisture content (b) at different points in time corresponding to the lines 1—25 sec., 2—50 sec., 3—100 sec. 4—200 sec., 5—400 sec., 6—800 °C; dependence of movable boundary coordinates x(s) (c) and the average moisture content in the layer SINT (d) on the time for T0 = 20°C, Sl0 = 0.5, Te = 160°C, kve = 0 (dry vapor). The dashed line represents the self-similar solution.

Figure 3b shows that with increasing moisture content, the drying time increases, the difference between the solutions in the final layer and self-similar one becomes more significant. It is caused by the fact that the proportion of the heat spent on heating the medium increases with the initial moisture content. It should be noted that with increasing initial moisture content, 'live' porosity in humid area decreases, and therefore the phase transitions in it are practically terminated realized; front evaporation scheme on a moving boundary is formed. With increasing moisture vapor at the outer boundary (Fig. 3c), the drying time of the layer at $k_{ve} \approx 1$becomes longer than in a self-similar formulation.

It is caused by a decrease in heat flux from the outside, which is determined by the temperature gradient in the dry zone. Figure 3d shows a similar dependence on the initial temperature T_0. The graphs show that the final drying of the finite thickness material and the temperature deep in it depend slightly on the starting temperature by the drying out time. This is due to the influence of two opposing factors. Increasing the initial temperature T_0 on the one hand leads to a greater heat content layer, and on the other hand—to a smaller temperature gradient in the dry zone, which determines the intensity of heat transfer from the outside.

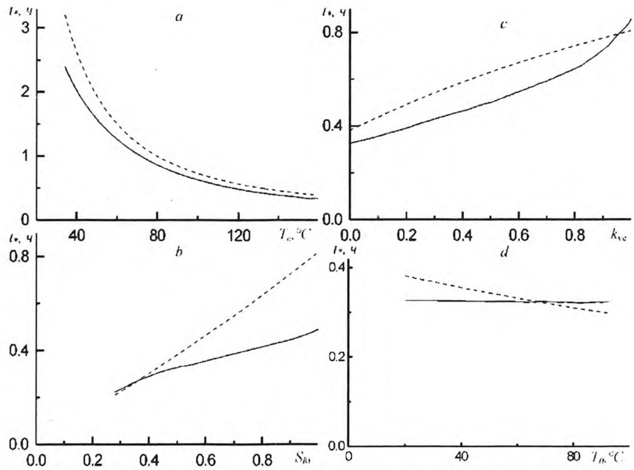

Fig. 3. Dependence of the drying time t * on the outer temperature Te (a), on the initial moisture content Sl0 (b), on vapor concentration kve (c) at the outer edge (x=0), on the initial temperature T0 (d). When this constant parameters take the following values: T0 = 20°C, Te = 160°C, kve = 0, Sl0 = 0.5.

Self-similar solution is shown by dotted lines.

Conclusions. It has been established that drying time of a thin layer is generally less than the formation of dry zones of the same size in an unbounded medium, since the heat supplied from the outer boundary is used to a greater extent for evaporation and in a less degree—for heating of the material. This pattern is broken for two cases: if vapor concentration at the outer boundary is close to 1, then in a humid area high temperature is set, which reduces the heat flow from the outside. If the initial temperature of the medium is close to the boiling point, then the supply of heat from the depth stops immediately.

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