

# Universal Optimization Algorithm for Gas-condensate Gathering

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**ABSTRACT**— *in this article we consider a solution of the condensate gathering optimization task when developing the gas-condensate field. As a feature of production control mathematical models appear nonlinear systems of equations that are irresolvable in an explicit form. This complicates the solutions of the optimization tasks significantly. In this article the optimization task was considered in the algebraic equations system being solved with an iterative method. The dynamic programming method was proposed to find out of optimal discrete choke diameters set, that provides the maximal condensate gathering with a restriction on gas production. In this work we evaluate the time advantage for discrete optimization task solution with the proposed method in comparison with the brute force method.*

**Keywords**— Dynamic programming, gas-condensate, optimization task

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## 1. INTRODUCTION

Production at gas-condensate fields has a distinctive feature related to the phase transition or condensation of heavy hydrocarbon fractions. The condensation process is possible with the pressure less than the dew point pressure. One of the tasks concerning gas-condensate mixture production is the exclusion or the maximum reduction of the stratum condensation. Herewith, the gas mixture is impoverished and the condensate production volumes decrease. To solve this problem it is necessary to maintain the production at the highest possible pressure in the stratum system. However, in this case, the production volume is reduced. For the purpose of wellhead control chokes with different bore diameters are applied. That is why it is more convenient to relate well flow rates and pressures to a choke diameter. The choke diameter increase causes a monotonic flow rate increase of both gas and condensate attending with the pressure reduction at wellbore and bottom-hole formation zone. Pressure reduction leads to the mixture impoverishment, i.e. the condensate production growth will be lower than the gas production growth. The monotonic dependence between the flow rates and choke diameters eliminates a meaningful statement of the optimization problem for the individual wells. When considering the total flow rate of all the wells we can see that the value of the final condensate production depends on the wells modes (chokes diameters). This allows us to define the optimization problem of condensate production maximizing for the given level of gas production. If wells are completely identical in all respects, the most effective production will be possible with the equal chokes diameters, flow rates and pressures as well. This is a direct consequence of the negative second derivative relationship between the condensate and gas flow rates. In practice, all wells have individual characteristics, associated with the location, stratum pressure in flow area and period of operation. Under these circumstances, finding the optimal solutions seems to be a trivial task.

## 2. MATHEMATICAL MODEL OF GAS-CONDENSATE PRODUCTION

During gas and condensate production the flow appears first in stratum then in well, choke and gas collection system. The equations describing the flow in these areas are sufficiently described in the literature [1, 6]. Generally they represent non-linear partial differential equations with variable coefficients and are solved only numerically with the finite difference method. Using the properties averaging procedure, the initial system of differential equations can be integrated, being reduced to an algebraic form. However, the resulting system is nonlinear and implicitly resolved regarding gas flow rate. The system of equations is written in the paper [4] as well the analytical solution of the problem of gas and condensate flow rates in implicit form. The system of equations relating production hydraulic parameters of n-wells in this case is as follows:

$$\begin{aligned}
 P_{ri}^2 - P_{bhi}^2 &= A_i Q_i + B_i Q_i^2 \\
 P_{bhi}^2 - P_{thi}^2 e^{2S} &= \theta_i Q_i^2 \\
 P_{thi}^2 - P_{li}^2 &= B_{chokei} Q_i^2 \\
 Q_{Ci} &= Q_i OGR_i(P_{bhi}) \\
 i &= 1, 2, \dots, n
 \end{aligned}$$

where  $i$  is the well number,  $Q_i$  is the gas flow rate,  $Q_{ci}$  is the condensate flow rate,  $OGR_i(P_{bhi})$  is the gas-condensate relationship depending on the pressure, mixture temperature and composition,  $P_{ri}$  is the stratum pressure,  $P_{bhi}$  is the bottom-hole pressure,  $P_{thi}$  is the wellhead pressure,  $P_{li}$  is the linear pressure at the choke outlet,  $A_i$  and  $B_i$  are the filtration resistances,  $\theta_i$  is the integral coefficient of the borehole hydraulic resistance,  $B_{chokei}$  is the integral coefficient of choke hydraulic resistance. The index shows the belonging to the  $i^{th}$  well.

The coefficients of the given system depend on gas flow rate and pressures in the system. Considering that we are unable to solve such system in an explicit way, we can solve it with the iteration method. For this reason it is also impossible to calculate the first derivatives explicitly, that greatly complicates the extremums search with the conventional methods.

Taking into account the fact that chokes diameters, determining the coefficients values are represented as a discrete set of numbers, the optimization problem appears to be a discrete programming problem.

### 3. STATEMENT OF THE PROBLEM AND ALGORITHM

In this article we consider the optimization task of gas-condensate gathering from a set of wells with the given gas flow rate total value. The gas flow rate volume is restricted by several causes: the capacity of transport network mainly, the storage tanks volumes and the time of raw materials processing.

According to the proposed mathematical model, every well ( $s$  from the set of all wells  $S$ ) can be presented by a couple of functions:

- $Q_s(d)$  defines gas flow rate per day for a well  $s$  with a fixed diameter  $d$ ;
- $C_s(d)$  defines gas-condensate flow rate per day for a well  $s$  with fixed diameter  $d$  (similarly to  $Q_{Ci}$  in the proposed mathematical model);

If there are high-producing wells with large gas and condensate flow rates among the others, it is reasonable to develop them in particular, ensuring herewith an increase of condensate production. However, we should take into account that wells development (increasing of  $d_i$ ) will lead to the decrease of bottom-hole pressure  $P_{bhi}$  that will result in decreasing of the gas-condensate relationship  $OGR_i(P_{bhi})$  so that the  $Q_{ci}$  condensate flow rate growth will be lower than condensate flow rate growth. In this conditions involving wells with the higher  $OGR_i(P_{bhi})$  values will be more profitable.

Using the described mathematical model we can determine the values of functions  $Q$  and  $C$  for every well. The original problem can also be presented as a non-linear programming task. There are various methods which can be applied to solve this task such as the genetic algorithm in particular, which, however, allow finding a local optimal solution that may not be global. The application of this method is complicated by the fact that the choke diameter at each borehole can take a value from a limited set of numbers. Herewith this task appears to be a discrete optimization problem.

Abstracting away from any mathematical model and introducing the functions  $Q$  and  $C$  as black boxes, let us we consider the original problem as a knapsack problem [5]. In the knapsack problem someone should collect a certain number of things with a maximum total cost for a given volume of the knapsack. The volume and cost of every thing are known as well [3]. In the original problem we consider a gas gathering network, which "volume" is limited by the maximum total gas collection, as a knapsack. Each well with the defined diameter can be considered as a thing which value is determined with the function  $C$ , and volume – with the function  $Q$ . The difference between the given problem and the knapsack problem is that we should choose only one thing, i. e. one well, so that it is not possible to give two different choke diameters for one well.

### 4. SOLUTION ALGORITHM FOR THE PRESENTED PROBLEM

To solve the problem we use the method based on dynamic programming. The dynamic programming method was first presented by Bellman in 1960 in his work Dynamic programming [2]. To solve the task using the dynamic programming method we need to get the optimal substructure of the problem, i.e. the optimal solution should contain the optimal solutions of auxiliary subtasks.

In the problem of optimal choke diameters set searching every well represents a class, where every instance of a class is presented by one diameter. We have to choose one instance from every class so that the total flow rate will be equal to

the predetermined constant and the condensate volume will be as large as possible.

Now, let us try to define the optimal substructure of the given problem. Every well is being operated conventionally independently of the other wells, i.e. the effect of gas production at the moment doesn't influence the production of other well significantly. Therefore, wells can be observed in any order  $p_1, p_2, \dots, p_n$  (where  $n$  is the total number of wells). We remind that  $Q_p(d)$  is the production volume of gas per day from wells with the number  $p$  when the diameter is fixed and equals  $d$ . Finally we have to determine the maximum condensate volume for the given production volume  $Q_{const}$ .

Let  $F$  be the optimization function, then  $F(Q_{const}, D) \rightarrow MAX$  when  $D$  is the solution vector.  $F = \sum_{p \in P} (C_p(d_p))$ , where  $C$  is the function that determines the condensate volume per day gathered from the well number  $p$  with the diameter  $d_p$  ( $P$  is a set of all wells). As well as each well can have the only one diameter we will consider the following substructure of the optimal problem: Let  $A(i, Q')$  be a problem solution if gas is gathered from the first  $i$  wells (in any fixed order) with the total flow rate volume  $Q'$ . So, the solution of the given task can be presented as the following:

$$A(i, Q') = MAX_{d \in D} (A(i-1, Q' - Q(p_i, d_{pi})) + C(p_i, d_{pi})) \quad (1)$$

#### 4.1 Proof of the Algorithm Correctness

THEOREM 1. The solution of  $A(i, Q)$  is optimal.

PROOF. Let us prove it by induction from  $i$ . It is obvious that for  $A(1, Q(p_1, d_{p1})) = C(p_1, d_{p1})$  all the solutions will be optimal (as they are the only ones). Let the solution for  $j : A(j, Q)$  be optimal. Let us prove the correctness for  $j + 1$ . Let us suppose that the solution calculated by the relation (1) is not optimal. Then there is a solution  $B > A(j+1, Q)$ . Then  $B > MAX_{d \in D} (A(j, Q' - Q(p_j, d_{pj})) + C(p_j, d_{pj}))$ , let  $d = d_m$  indicates the  $d$  whereby the relation  $A(j, Q' - Q(p_j, d_{pj})) + C(p_j, d_{pj})$  is maximal, then  $B > A(j, Q' - Q(p_j, d_{pj})) + C(p_j, d_{pj})$ , this means that there are the set of diameters  $d'_1, d'_2, \dots, d'_j$  (which are the solutions for  $B$ ), so  $C(p_1, d'_{p1}) + \dots + C(p_j, d'_{pj}) = B$  if  $Q(p_1, d'_{p1}) + \dots + Q(p_j, d'_{pj}) = Q$ . There are diameter sets  $d_1, d_2, \dots, d_j$  (which are the solutions for  $A(j, Q)$ ), so that  $C(p_1, d_{p1}) + \dots + C(p_j, d_{pj}) = A$ . However, by the induction hypothesis there are some  $d_1, d_2, \dots, d_{j-1}$ , so that  $Q(p_1, d_{p1}) + \dots + Q(p_{j-1}, d_{p_{j-1}}) + Q(p_j, d'_{pj}) = Q$ , thus they are the optimal solution. So in the problem with the solution  $B$  there are the diameters  $C(p_1, d'_{p1}) + C(p_2, d'_{p2}) + \dots + C(p_{j-1}, d'_{p_{j-1}}) > C(p_1, d_{p1}) + C(p_2, d_{p2}) + \dots + C(p_{j-1}, d_{p_{j-1}})$ , what contradicts the induction hypothesis, or  $C(p_j, d'_{pj}) > C(p_j, d_{pj})$  if  $Q(p_j, d'_{pj}) = Q(p_j, d_{pj})$ , what is also a contradiction, because one well cannot produce the same flow rate with two different diameters.

#### 4.2 valuation of the Algorithm Complexity

To calculate the function  $A(i, Q)$  values we need to precalculate all the values  $A(i-1, Q_a)$  for every  $Q_a \leq Q_{const}$ . To calculate the values the following algorithm can be used:

1) For all  $p_i \in P$

a) For all integer  $Q_a \leq Q_{const}$

i) For all  $d \in D$

(a) if  $A(i, Q_a) \leq A(i-1, Q_a - Q(p_i, d)) + C(p_i, d)$  then

$$A(i, Q_a) = A(i-1, Q_a - Q(p_i, d)) + C(p_i, d)$$

Thus, calculating the function  $A(i, Q)$  values with this method it is necessary to perform  $|P| * |D| * |Q_{const}|$  operations. This is clear from the fact that the cycles in the steps 1, (a) and i are performed without interruptions. Therefore, the computational complexity of the presented method is proportional to  $O(|P| * |D| * |Q_{const}|)$ .

### 5. CONCLUSION

This article presents the mathematical model of gas-condensate gathering from one well as well as the versatile maximization algorithm of gas-condensate gathering from multiple wells with a given total number of gas units to be collected (thous. cub. m. per day). The developed algorithm solves the presented problem efficiently in time proportional to the product of the wells number on the total flow rate and the number of different diameters. In future research we will consider well cluster. This is mean that each well effect to each other well.

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